

ALGEBRA II CURRICULUM
(August, 2007)

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COURSE DESCRIPTION

Algebra II is designed to reinforce the concepts already learned in Algebra I and to expand those topics. Algebra II also introduces new more advanced concepts. This course will prepare students to proceed to Geometry CP and Pre-Calculus.

Topics included in this course are functions, explicit and recursive formulas, variation and graphs, systems, matrices, quadratic equations, powers, and polynomials. The course makes use of both scientific and graphing calculators and focuses on theory and application.

CORE CURRICULUM CONTENT STANDARDS

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS)

All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

STANDARD 4.2 (GEOMETRY AND MEASUREMENT)

All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

STANDARD 4.3 (PATTERNS AND ALGEBRA)

All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)

All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

STANDARD 4.5 (MATHEMATICAL PROCESSES)

All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

STANDARD 8.1 (COMPUTER AND INFORMATION LITERACY)

All students will use computer applications to gather and organize information and to solve problems.

STANDARD 8.2 (TECHNOLOGY EDUCATION)

All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual, society, and the environment.

CUMULATIVE PROGRESS INDICATORS

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS)

A. Number Sense

1. Extend understanding of the number system by constructing meanings for the following (unless otherwise noted, all indicators for grade 8 pertain to these sets of numbers as well):
 - Rational numbers
 - Exponents
 - Roots
 - Absolute values
 - Numbers represented in scientific notation
2. Understand and use ratios, proportions, and percents (including percents greater than 100 and less than 1) in a variety of situations.
3. Compare and order numbers of all named types.
4. Extend understanding of the number system to all real numbers.
5. Compare and order rational and irrational numbers.

B. Numerical Operations

1. Use and explain procedures for performing calculations involving addition, subtraction, multiplication, division, and exponentiation with integers and all number types named above with:
 - Pencil-and-paper
 - Mental math
 - Calculator
2. Use exponentiation to find whole number powers of numbers.
3. Find square and cube roots of numbers and understand the inverse nature of powers and roots.
4. Solve problems involving proportions and percents.
5. Understand and apply the standard algebraic order of operations, including appropriate use of parentheses.
6. Extend understanding and use of operations to real numbers and algebraic procedures.
7. Develop, apply, and explain methods for solving problems involving rational and negative exponents.
8. Perform operations on matrices.
 - Addition and subtraction
 - Scalar multiplication
9. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

STANDARD 4.2 (GEOMETRY AND MEASUREMENT)

A. Geometric Properties

1. Understand and apply concepts involving lines.
 - Bisectors and perpendicular bisectors
 - Parallel, perpendicular, and intersecting planes
2. Understand and apply the Pythagorean theorem.
3. Understand and apply the concept of similarity.
 - Using proportions to find missing measures

B. Transforming Shapes

1. Understand and apply transformations.
 - Finding the image, given the pre-image, and vice-versa
 - Sequence of transformations needed to map one figure onto another
 - Reflections, rotations, and translations result in images congruent to the pre-image
 - Dilations (stretching/shrinking) result in images similar to the pre-image

C. Coordinate Geometry

1. Use coordinates in four quadrants to represent geometric concepts.
2. Use a coordinate grid to model and quantify transformations (e.g., translate right 4 units).
3. Use coordinate geometry to represent and verify properties of lines.
 - Distance between two points
 - Midpoint and slope of a line segment
 - Finding the intersection of two lines
 - Lines with the same slope are parallel
 - Lines that are perpendicular have slopes whose product is -1

D. Units of Measurement

1. Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation.
2. Solve problems requiring calculations that involve different units of measurement within a measurement system (e.g., $4'3''$ plus $7'10''$ equals $12'1''$).
3. Use approximate equivalents between standard and metric systems to estimate measurements (e.g., 5 kilometers is about 3 miles).
4. Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation.
5. Recognize that all measurements of continuous quantities are approximations.
6. Solve problems that involve compound measurement units, such as speed (miles per hour), air pressure (pounds per square inch), and population density (persons per square mile).

STANDARD 4.3 (PATTERNS AND ALGEBRA)

A. Patterns

1. Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers.
 - Descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions
 - Finite and infinite sequences
 - Arithmetic sequences (i.e., sequences generated by repeated addition of a fixed number, positive or negative)
 - Geometric sequences (i.e., sequences generated by repeated multiplication by a fixed positive ratio, greater than 1 or less than 1)
 - Generating sequences by using calculators to repeatedly apply a formula
2. Use models and algebraic formulas to represent and analyze sequences and series.
 - Explicit formulas for n th terms
 - Sums of finite arithmetic series
 - Sums of finite and infinite geometric series
3. Develop an informal notion of limit.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.
 - Slope of a line or curve
 - Domain and range
 - Intercepts
 - Continuity
 - Maximum/minimum
 - Estimating roots of equations
 - Intersecting points as solutions of systems of equations
 - Rates of change
3. Understand and perform transformations on commonly-used functions.
 - Translations, reflections, dilations
 - Effects on linear and quadratic graphs of parameter changes in equations
 - Using graphing calculators or computers for more complex functions
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.
 - Linear vs. non-linear
 - Symmetry
 - Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.
 - Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
 - Direct and inverse variation
 - Absolute value
 - Expressions, equations and inequalities
 - Same function can model variety of phenomena
 - Growth/decay and change in the natural world
2. Analyze and describe how a change in an independent variable leads to change in a dependent one.
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Solve simple linear equations informally, graphically, and using formal algebraic methods.
 - Multi-step, integer coefficients only (although answers may not be integers)
 - Using paper-and-pencil, calculators, graphing calculators, spreadsheets, and other technology
2. Solve simple linear inequalities.
3. Create, evaluate, and simplify algebraic expressions involving variables.
 - Order of operations, including appropriate use of parentheses
 - Distributive property
 - Substitution of a number for a variable
 - Translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality, and vice versa
4. Understand and apply the properties of operations, numbers, equations, and inequalities.
 - Additive inverse
 - Multiplicative inverse
 - Addition and multiplication properties of equality
 - Addition and multiplication properties of inequalities
5. Evaluate and simplify expressions.
 - Add and subtract polynomials
 - Multiply a polynomial by a monomial or binomial
 - Divide a polynomial by a monomial
6. Select and use appropriate methods to solve equations and inequalities.
 - Linear equations – algebraically
 - Quadratic equations – factoring (when the coefficient of x^2 is 1) and using the quadratic formula
 - All types of equations using graphing, computer, and graphing calculator techniques
7. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)

A. Data Analysis

1. Evaluate the use of data in real-world contexts.
 - Accuracy and reasonableness of conclusions drawn
 - Bias in conclusions drawn (e.g., influence of how data is displayed)
 - Statistical claims based on sampling
2. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.

B. Probability

1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
2. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these 3. Determine probabilities in complex situations.
 - Conditional events
 - Complementary events
 - Dependent and independent events
3. Estimate probabilities and make predictions based on experimental and theoretical probabilities.

C. Discrete Mathematics—Systematic Listing and Counting

1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students).
2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.
3. Justify solutions to counting problems.

STANDARD 4.5 (MATHEMATICAL PROCESSES)

By the end of Grade 12, students will:

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - Open-ended problems
 - Non-routine problems
 - Problems with multiple solutions
 - Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
 - Reading and writing
 - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
 - Counterexamples as a means of disproving conjectures
 - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
 - Concrete representations (e.g., base-ten blocks or algebra tiles)

- Pictorial representations (e.g., diagrams, charts, or tables)
 - Symbolic representations (e.g., a formula)
 - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
 3. Use representations to model and interpret physical, social, and mathematical phenomena.

STANDARD 8.1 (COMPUTER AND INFORMATION LITERACY)

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.

STANDARD 8.2 (TECHNOLOGY EDUCATION)

1. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
2. Use computer software to make and verify conjectures about geometric objects.
3. Use computer-based laboratory technology for mathematical applications in the sciences.

SUGGESTED ACTIVITIES

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS)

Ratios & Proportions

Solve problems involving ratios and solve proportion problems.

In-class Activity: 2001 HSPA Examination, Problems 12, 15, 19, 27, 31

In-class Activity: PH 8-7 p. 370

STANDARD 4.2 (GEOMETRY AND MEASUREMENT)

Coordinate Geometry

Solve problems involving transformations; translations, reflections rotations. Review the properties of congruence and similarity.

In-class Activity: 2001 HSPA Examination, Problems 22, 28, 13

In-class Activity: PH 9-3 p. 398

Matrices

Investigate the use of matrices to perform transformations; size changes, scale changes, reflections, translations and rotations.

In-class Activity: Size Changes (A.A. 4-3, p. 220)

In-class Activity: Composites of Transformations (A.A. 4-6, p. 239)

In class Activity: PH Chapter 13 p.568-609

Right-Triangle Trigonometry

Investigate the properties of and relationship between the sine and cosine trigonometric ratios.

In-class Activity: Rotations, Sines, and Cosines (A.A. 10-3, p. 622)

In-class Activity: 2001 HSPA Examination, Problems 14, 17

In-class Activity: PH 17-1 p. 732

STANDARD 4.3 (PATTERNS AND ALGEBRA)

Sequences and Series

Investigate patterns, sequences and series.

In-class Activity: Patterns and Sequences (A.A. 1-6, p. 41)

In-class Activity: 2001 HSPA Examination, Problems 3, 11, 13, 16, 21, 24, 34, 36

In-class Activity: PH 14-1 p. 615

Variation

Investigate the characteristics of variation.

In-class Activity: Functions of Variation (A.A. 2-2, p. 83)

In-class Activity: PH 6-9 p. 283

Variation

Investigate inverse variation using a graphing calculator.

In-class Activity: Introduction to Automatic Graphers (A.A. 2-4, p. 94)

In-class Activity: Introduction to Automatic Graphers (A.A. 2-5, p. 103)

In-class Activity: PH 6-9 p. 283

Matrices

Investigate the properties of the inverses of matrices.

In-class Activity: Matrices and Inverses (A.A. 5-4, p. 298)

In-class Activity: PH 13-5 p. 591

Transformations

Investigate translations of quadratic functions.

In-class Activity: Graphs and Translations (A.A. 6-2, p. 356)

In-class Activity: PH 9-2 p. 393

Solutions to Equations

Investigate the number and nature of solutions to quadratic equations.

In-class Activity: Predicting the Number of Real Solutions to a Quadratic Equation (A.A. 6-9, p. 399)

In-class Activity: 2001 HSPA Examination, Problems 9-11, 20, 21, 23

In-class Activity: PH 8-4 p. 357

Functions & Relationships

Investigate the properties and behavior of functions.

In-class Activity: 2001 HSPA Examination, Problems 7, 9, 20, 23

In-class Activity: PH 3-3 p. 119

Power Functions

Investigate the behavior of power functions with rational exponents.

In-class Activity: Non-integer Power Functions (A.A. 7-6, p. 457)

In-class Activity: PH 12-2 p. 525

Exponential Functions Logarithmic Functions

Investigate the behavior of base e and the natural log..

In-class Activity: The number e (A.A. 9-2, p. 546)

In-class Activity: PH 12-4 p. 532

Right-Triangle Trigonometry

Investigate the relationship between angles and sides of a triangle.

In-class Activity: The Law of Sines (A.A. 10-8, p. 652)

In-class Activity: PH 17-1 p. 732

Solutions to Polynomial Equations

Investigate the solutions of polynomial equations.

In-class Activity: Factors and Graphs (A.A.11-4, p. 698)

In-class Activity: 2001 HSPA Examination, Problems 2, 10, 24, 29, 35

In-class Activity: PH 11-1 p.483

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)

Linear Models

Investigate the use of linear models utilizing graphing calculators to approximate data.

In-class Activity: Introduction to Automatic Graphers (A.A. 3-5, p. 168)

In-class Activity: PH 16-1 p. 692

STANDARD 4.5 (MATHEMATICAL PROCESSES)

Order of Operations with Polynomials

Investigate order of operations on the calculator.

In-class Activity: Grouping Symbols and Calculators (A.A. 1-2, p. 18)

In-class Activity: 2001 HSPA Examination, Problems 9, 11, 14, 20, 23

In-class Activity: PH 5-2 p. 212

Radical Functions

Investigate the graphs of radical functions.

In-class Activity: Graphs of Radical Functions. (A.A. 8-6, p. 510)

In-class Activity: PH 9-1 p. 389

Right-Triangle Trigonometry

Investigate the graphs of the trigonometric functions.

In-class Activity: Graphing $y = \cos x$. (A.A. 10-7, p. 646)

In-class Activity: PH 17-5 p. 760

Standard 8.1 (COMPUTER AND INFORMATION LITERACY)

Fit an exponential model to data.

In-class Activity: A.A. Computer Master 14

Standard 8.2 (TECHNOLOGY EDUCATION)

Generate tables with explicit and recursive formulas.

In-class Activity: A.A. Computer Master 1

INSTRUCTIONAL STRATEGIES

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS)

- Provide a brief but comprehensive review of the Real Number System (the sets of Natural Numbers, Integers, Rational Numbers and Irrational Numbers, identities and inverses).
- Provide practice converting numbers to different forms (fractions, decimals, exponents and radicals). Use calculators to illustrate the representation of numbers in these different forms.
- Provide practice for the rules governing the order of operations. Use calculators to illustrate the rules.
- Provide a comprehensive and thorough treatment of ratio and proportion, focusing on solving proportion problems.
- Provide instruction on operations and involving matrices. Represent real-world problems in matrix form and provide practice solving such systems.

STANDARD 4.2 (GEOMETRY AND MEASUREMENT)

- Provide instruction on transformations (translations and rotations).
- Provide instruction on the properties of geometric constructs (circles and other conic sections, and polygons).
- Provide instruction on the properties of trigonometric ratios.
- Provide instruction on vectors and represent vectors in matrix form. Apply distance formula.
- Extensively employ graphing calculators to illustrate the concepts.

STANDARD 4.3 (PATTERNS AND ALGEBRA)

- Provide instruction on the algebra of vectors. Represent vectors as matrices. Apply distance formula.
- Provide instruction on trigonometric ratios and provide applications involving vectors.
- Provide instruction on sequences and series with applications. Introduce the concept of the limit of a sequence. Use graphing calculators to illustrate concepts.
- Provide instruction on functions and polynomials to include linear, linear systems and quadratic; the roots of equations and inequalities and graphs. Extensively employ graphing calculators to illustrate concepts.
- Model real world problems focusing on applications in mathematics, science and economics. Extensively employ graphing calculators and introduce EXCEL as a modeling tool. Assign appropriate projects to reinforce learning.

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)

□ Provide instruction introducing students to utilizing data to make projections (linear and quadratic best fit). Use graphing calculators to find an optimal fit and make projections.
Review characteristics of a linear fit and discuss limitations.

STANDARD 4.5 (MATHEMATICAL PROCESSES)

□ Provide instruction on strategies and procedures for solving multi-step, open-ended problems involving mathematical concepts previously presented. Provide numerous examples including those drawn from 2001 HSPA Examination (see ACTIVITIES). Allow students to develop their own problems with a corresponding scoring rubric for presentation to the class. Utilize both individual and group projects.

□ Provide students the opportunity to communicate findings on open-ended problems; written analyses, extemporaneous analyses and formal presentations.

STANDARD 8.1 (COMPUTER AND INFORMATION LITERACY)

- Provide instruction on how to create a simple spreadsheet, enter data and interpret information.
- Choose appropriate tools and information resources to support research and solve real world problems.

STANDARD 8.2 (TECHNOLOGY EDUCATION)

Describe how variations in resources can affect solutions to a technological problem

EVALUATION/ASSESSMENT OF STUDENTS

Assessment should consist of the following:

□ regular and frequent quizzes comprised of a variety of problem types ranging from multiple choice to open-ended problems.

□ periodic, comprehensive examinations comprised of a variety of problem types ranging from multiple choice to open-ended problems.

□ Regular projects involving applications of mathematical concepts extensively utilizing technology;

- graphing calculators and computer software (EXCEL).

RESOURCES

- PH = Algebra II with Trigonometry – Prentice Hall
- AA = ADVANCED ALGEBRA – UCSMP – (The University of Chicago School Mathematics Project)
- TI 83 or TI 89 Graphing Calculators
- 2001 HSPA Examination

□ Microsoft Windows Software Programs (EXCEL, WORD, POWERPOINT)