

ALGEBRA II CONCEPTS

August 2009

COURSE DESCRIPTION:

Algebra II Concepts is designed for those students who plan to attend college and require additional background in Algebra concepts. This course will enable students to continue their mastery of skills introduced in Algebra I. Students will apply learned skills to Algebra II topics and related world problems. Students completing this course will have mastered topics necessary for successful transition into geometry and other math electives. Graphing calculators will be utilized throughout the course while covering the four main content areas: solving equations, graphing, exponents, and quadratics.

CORE CURRICULUM CONTENT STANDARDS:

STANDARD 4.1 (Number and numerical operations) All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

STANDARD 4.2 (Geometry and measurement) All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

STANDARD 4.3 (Patterns and algebra) All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

STANDARD 4.4 (Data analysis, probability, and discrete mathematics) All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

STANDARD 4.5 (Mathematical processes) All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

STANDARD 8.1 (Computer and information literacy) All students will use computer applications to gather and organize information and to solve problems.

STANDARD 8.2 (Technology Education) All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual, society, and the environment.

CUMULATIVE PROGRESS INDICATORS:

STANDARD 4.1 (Number and numerical operations)

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Perform operations on matrices.
 - ♣ Addition and subtraction
 - ♣ Scalar multiplication
3. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

STANDARD 4.2 (Geometry and measurement)

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.
 - ♣ Distance between two points
 - ♣ Midpoint and slope of a line segment
 - ♣ Finding the intersection of two lines
 - ♣ Lines with the same slope are parallel
 - ♣ Lines that are perpendicular have slopes whose product is -1

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
 - ♣ Degree of accuracy of a given measurement tool

STANDARD 4.3 (Patterns and algebra)

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.
 - ♣ Explicit formulas for n^{th} terms
 - ♣ Sums of finite arithmetic series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.
 - ♣ Slope of a line or curve
 - ♣ Domain and range
 - ♣ Intercepts
 - ♣ Continuity
 - ♣ Maximum/minimum
 - ♣ Estimating roots of equations
 - ♣ Intersecting points as solutions of systems of equations
 - ♣ Rates of change
3. Understand and perform transformations on commonly-used functions.
 - ♣ Effects on linear and quadratic graphs of parameter changes in equations
 - ♣ Using graphing calculators or computers for more complex functions
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.

- ♣ Linear vs. non-linear
- ♣ Symmetry

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.
 - ♣ Linear, quadratic, exponential, and periodic (sine and cosine)
 - ♣ Direct and inverse variation
 - ♣ Absolute value
 - ♣ Expressions, equations and inequalities
 - ♣ Same function can model variety of phenomena
 - ♣ Applications in mathematics, biology, and economics (including compound interest)

2. Analyze and describe how a change in an independent variable leads to change in a dependent one.

D. Procedures

1. Evaluate and simplify expressions.
 - ♣ Add and subtract polynomials
 - ♣ Multiply a polynomial by a monomial or binomial
 - ♣ Divide a polynomial by a monomial

2. Select and use appropriate methods to solve equations and inequalities.
 - ♣ Linear equations - algebraically
 - ♣ Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula
 - ♣ All types of equations using graphing, computer, and graphing calculator techniques

3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

STANDARD 4.4 (Data analysis, probability, and discrete mathematics)

A. Data Analysis (or Statistics)

1. Evaluate the use of data in real-world contexts.
 - ♣ Accuracy and reasonableness of conclusions drawn
 - ♣ Bias in conclusions drawn (e.g., influence of how data is displayed)
 - ♣ Statistical claims based on sampling

2. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.

B. Probability

1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
2. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.
3. Determine probabilities in complex situations.
 - ♣ Conditional events
 - ♣ Complementary events
 - ♣ Dependent and independent events

STANDARD 4.5 (Mathematical processes)

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - ♣ Open-ended problems
 - ♣ Non-routine problems
 - ♣ Problems with multiple solutions
 - ♣ Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.

4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
 - ♣ Reading and writing
 - ♣ Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).

6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
 - ♣ Counterexamples as a means of disproving conjectures
 - ♣ Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
 - ♣ Concrete representations (e.g., base-ten blocks or algebra tiles)
 - ♣ Pictorial representations (e.g., diagrams, charts, or tables)
 - ♣ Symbolic representations (e.g., a formula)
 - ♣ Graphical representations (e.g., a line graph)

2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences (cf. science standards).

STANDARD 8.1 (Computer and information literacy)

A. Basic Computer Skills and Tools

1. Construct a spreadsheet, enter data, use mathematical or logical functions to manipulate and process data, generate charts and graphs, and interpret the results.
2. Given a database, define fields, input data from multiple records, produce a report using sort and query, and interpret the data.

3. Discuss and/or demonstrate the capability of emerging technologies and software in the creation of documents or files.

B. Application of Productivity Tools

Social Aspects

1. Describe the potential and implications of contemporary and emerging computer applications for personal, social, lifelong learning, and workplace needs.
2. Exhibit legal and ethical behaviors when using information and technology, and discuss consequences of misuse.

Information Access and Research

5. Select and use specialized databases for advanced research to solve real world problems.
6. Identify new technologies and other organizational tools to use in personal, home, and/or work environments for information retrieval, entry, and presentation.
7. Evaluate information sources for accuracy, relevance, and appropriateness.

Problem Solving and Decision Making

9. Create and manipulate information, independently and/or collaboratively, to solve problems and design and develop products.
10. Identify, diagnose, and suggest solutions for non-functioning technology systems.
11. Identify a problem in a content area and formulate a strategy to solve the problem using brainstorming, flowcharting, and appropriate resources.

12. Integrate new information into an existing knowledge base and communicate the results in a project or presentation.

STANDARD 8.2 (Technology Education)

A. Nature and Impact of Technology

1. Provide various examples of how technological developments have shaped human history.

B. Design Process and Impact Assessment

1. Evaluate the function, value, and appearance of technological products, systems, and environments from the perspective of the user and the producer.
2. Develop methods for creating possible solutions, modeling and testing solutions, and modifying proposed design in the solution of a technological problem using hands-on activities.

SUGGESTED ACTIVITIES THAT ADDRESS THESE STANDARDS MAY INCLUDE BUT ARE NOT LIMITED TO:

STANDARD 4.1 (Number and numerical operations):

- A. (AA p.15 #10-14) Solve problems involving classifying sets of numbers as real, rational, irrational, etc.

- B. (PH section 1-7 to 1-9) Problems involving the laws of exponents and simplifying expressions involving exponents.
- C. (PH section 3-5) Solve problems by graphing a function and estimating the values of y when x is an irrational number.

STANDARD 4.2 (Geometry and measurement)

- C. 1. (AA p.146-151) Determining whether lines are parallel, perpendicular, or neither based on slopes.
- 2. (PH section 10-1) Find the distance and midpoint between two points using the coordinates of each point and the appropriate formulas.
- D. (PH section 1-9) Solve problems using scientific notation (ex: Converting from scientific to standard notation and vice versa)

STANDARD 4.3 (Patterns and algebra)

- A. (AA sections 1-7 to 1-9) Writing explicit and recursive formulas for arithmetic sequences.
- B. (PH section 9-4) Finding the vertices, max/min, and continuity of parabolas.
- C. (AA sections 2-1 and 2-2) Solving direct and inverse variation problems that model real world phenomena.
- D. (PH sections 5-3 to 5-5) Using the FOIL method to both multiply and factor polynomials.

STANDARD 4.4 (Data analysis, probability, and discrete mathematics)

- A. (AA section 3-6) Finding the line of best fit for a scatter plot.
- B. (PH section 15-5) Determining how likely an event is to occur using both theoretical and experimental probability.

STANDARD 4.5 (Mathematical processes)

- A. (AA section 5-2) Finding solutions to systems of linear and non-linear equations where there are multiple solutions
- B. (AA section 1-1) Using appropriate mathematical terms to describe solutions to problems both orally and in written solutions.
- C. (AA sections 5-9 to 5-10) Connecting inequalities to linear programming to find maximum profits and minimum costs for business.
- D. (PH section 14-5) Proving statements about positive integers are true by using the principle of mathematical induction.
- E. (SAT p. 584) Representing a function in a variety of ways (table, equation, and graph).
- F. (AA section 7-1) Use graphing technology to study the characteristics of power functions.

STANDARD 8.1 (Computer and information literacy)

- A. Using graphing technology to create matrices that are used to solve linear systems.

- B. Use computer programming technologies to compute compound interest problems. (also with graphing calculators)

INSTRUCTIONAL STRATEGIES:

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS)

- Provide a brief but comprehensive review of the Real Number System (the sets of Natural Numbers, Integers, Rational Numbers and Irrational Numbers, identities and inverses).
- Provide practice converting numbers to different forms (fractions, decimals, exponents and radicals). Use calculators to illustrate the representation of numbers in these different forms.
- Provide practice for the rules governing the order of operations. Use calculators to illustrate the rules.
- Provide a comprehensive and thorough treatment of ratio and proportion, focusing on solving proportion problems.
- Provide instruction on operations and involving matrices. Represent real-world problems in matrix form and provide practice solving such systems

STANDARD 4.2 (GEOMETRY AND MEASUREMENT)

- Provide instruction on the properties of trigonometric ratios.
- Extensively employ graphing calculators to illustrate the concepts.
- Provide instruction on finding the distance between two points and the midpoint of a line segment.

STANDARD 4.3 (PATTERNS AND ALGEBRA)

- Provide instruction on sequences and series with applications. Introduce the concept of the limit of a sequence. Use graphing calculators to illustrate concepts.
- Provide instruction on functions and polynomials to include linear, linear systems and quadratic; the roots of equations and inequalities and graphs. Extensively employ graphing calculators to illustrate concepts.
- Model real world problems focusing on applications in mathematics, science and economics.

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)

- Provide instruction introducing students to utilizing data to make projections (linear and quadratic best fit). Use graphing calculators to find an optimal fit and make projections. Review characteristics of a linear fit and discuss limitations.

STANDARD 4.5 (MATHEMATICAL PROCESSES)

- Provide instruction on strategies and procedures for solving multi-step, open-ended problems involving mathematical concepts previously presented.
- Allow students to develop their own problems with a corresponding scoring rubric for presentation to the class. Utilize both individual and group projects.
- Provide students the opportunity to communicate findings on open-ended problems; written analyses, extemporaneous analyses and formal presentations.

STANDARD 8.1 (COMPUTER INFORMATION AND LITERACY)

- Provide instruction on how to input data into graphing technologies in order to solve systems of equations.
- Provide instruction on computer/graphing calculator programming to find compound interest.

EVALUATION/ASSESSMENT OF STUDENTS:

Assessment should consist of the following:

- Regular and frequent quizzes comprised of a variety of problem types ranging from multiple choice to open-ended problems.
- Periodic, comprehensive examinations comprised of a variety of problem types ranging from EVALUATION/ASSESSMENT OF STUDENTS multiple choice to open-ended problems.
- Regular projects involving applications of mathematical concepts extensively utilizing technology; graphing calculators.

EVALUATION/ASSESSMENT OF CURRICULUM:

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule (see attached)

RESOURCES/BIBLIOGRAPHY:

(AA)- Advanced Algebra (The university of Chicago School Mathematics Project)

(PH)- Prentice Hall: Algebra II with Trigonometry

(SAT)- College Board: The Official SAT Study Guide

TI-83 Graphing Calculators

Microsoft Windows Software programs (EXEL, WORD, POWERPOINT)