

AP CALCULUS AB

2006

Course Description

An AP course in calculus consists of a full high school academic year of work that is comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both, from institutions of higher learning.

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses covering the equivalent of four full years of high school mathematics before attempting calculus. These courses include algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. With a solid foundation in courses taken before AP, students will be well prepared to handle the rigor of a course at this level. Students who take an AP Calculus course should do so with the intention of lacing out of a comparable college calculus course. This may be done through the AP Exam, a college placement exam, or any other method employed by the college.

Topic Outline

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytical information and on the use of calculus both to predict and to explain The observed local and global behavior of a function.

Limits of functions(including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of their domain)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric consequences
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second Derivatives

- Corresponding characteristics of graphs f , f' , and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonic and concavity
- Optimization, both absolute(global) and relative(local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval
- Basic properties of definite integrals(examples include additivity and linearity)

Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance Traveled by a particle along a line.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables(including change of limits for definite integrals)

Applications of ant differentiation

- Finding specific ant derivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling(in particular, studying the equation $y' = ky$ and exponential growth)

Numerical approximations to definite integrals Use of Riemann sums(using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Core Curriculum Content Standards

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS) ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.

STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.

Cumulative Progress Indicators

Standard 4.1

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.

3. Perform operations on matrices.
 - Addition and subtraction
 - Scalar multiplication
4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

Standard 4.2

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Geometric Properties

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.
 - Parallel lines – transversal, alternate interior angles, corresponding angles
 - Triangles
 - a. Conditions for congruence
 - b. Segment joining midpoints of two sides is parallel to and half the length of the third side
 - c. Triangle Inequality
 - Minimal conditions for a shape to be a special quadrilateral
 - Circles – arcs, central and inscribed angles, chords, tangents
 - Self-similarity
4. Use reasoning and some form of proof to verify or refute conjectures and theorems.
 - Verification or refutation of proposed proofs
 - Simple proofs involving congruent triangles
 - Counterexamples to incorrect conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.
3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.
 - Fractals (e.g., Sierpinski's Triangle)
 - Patterns in areas and perimeters of self-similar figures
 - Outcome of extending iterative process indefinitely

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.
 - Distance between two points
 - Midpoint and slope of a line segment
 - Finding the intersection of two lines
 - Lines with the same slope are parallel
 - Lines that are perpendicular have slopes whose product is -1
2. Show position and represent motion in the coordinate plane using vectors.
 - Addition and subtraction of vectors

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
 - Degree of accuracy of a given measurement tool
 - Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.
 - Similar triangles
 - Pythagorean theorem
 - Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.
 - Approximation of area using grids of different sizes
 - Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
 - Estimation of area, perimeter, volume, and surface area

Standard 4.3

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.
 - Explicit formulas for n^{th} terms
 - Sums of finite arithmetic series
 - Sums of finite and infinite geometric series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.

- Slope of a line or curve
 - Domain and range
 - Intercepts
 - Continuity
 - Maximum/minimum
 - Estimating roots of equations
 - Intersecting points as solutions of systems of equations
 - Rates of change
3. Understand and perform transformations on commonly-used functions.
 - Translations, reflections, dilations
 - Effects on linear and quadratic graphs of parameter changes in equations
 - Using graphing calculators or computers for more complex functions
 4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.
 - Linear vs. non-linear
 - Symmetry
 - Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.
 - Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
 - Direct and inverse variation
 - Absolute value
 - Expressions, equations and inequalities
 - Same function can model variety of phenomena
 - Growth/decay and change in the natural world
 - Applications in mathematics, biology, and economics (including compound interest)
2. Analyze and describe how a change in an independent variable leads to change in a dependent one.
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Evaluate and simplify expressions.
 - Add and subtract polynomials
 - Multiply a polynomial by a monomial or binomial
 - Divide a polynomial by a monomial
2. Select and use appropriate methods to solve equations and inequalities.
 - Linear equations – algebraically
 - Quadratic equations – factoring (when the coefficient of x^2 is 1) and using the quadratic formula

All types of equations using graphing, computer, and graphing calculator techniques
3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Standard 4.4

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Data Analysis

1. Use surveys and sampling techniques to generate data and draw conclusions about large groups.
 - Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling)
2. Evaluate the use of data in real-world contexts.
 - Accuracy and reasonableness of conclusions drawn
 - Bias in conclusions drawn (e.g., influence of how data is displayed)
 - Statistical claims based on sampling
3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.
4. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.
5. Analyze data using technology, and use statistical terminology to describe conclusions.
 - Measures of dispersion: variance, standard deviation, outliers
 - Correlation coefficient
 - Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)

B. Probability

1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
2. Use concepts and formulas of area to calculate geometric probabilities.
3. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.
4. Determine probabilities in complex situations.
 - Conditional events
 - Complementary events
 - Dependent and independent events
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.
6. Understand and use the “law of large numbers” (that experimental results tend to approach theoretical probabilities after a large number of trials).

C. Discrete Mathematics—Systematic Listing and Counting

1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students).
2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.

3. Justify solutions to counting problems.
4. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability.

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms

1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems.
 - Circuits that include every edge in a graph
 - Circuits that include every vertex in a graph
 - Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring
 - Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures)
2. Explore strategies for making fair decisions.
 - Combining individual preferences into a group decision (e.g., determining winner of an election or selection process)
 - Determining how many Student Council representatives each class (9th, 10th, 11th, and 12th grade) gets when the classes have unequal sizes (apportionment)

Standard 4.5

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - Open-ended problems
 - Non-routine problems
 - Problems with multiple solutions
 - Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
 - Reading and writing
 - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a

- single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
 4. Apply mathematics in practical situations and in other disciplines.
 5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
 6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
 - Counterexamples as a means of disproving conjectures
 - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
 - Concrete representations (e.g., base-ten blocks or algebra tiles)
 - Pictorial representations (e.g., diagrams, charts, or tables)
 - Symbolic representations (e.g., a formula)
 - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences.

Suggested activities that address these standards may include but are not limited to:

Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the

connections among these representations. (NJCCCS 4.2, 4.5)

Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems. (NJCCCS 4.1, 4.5)

Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems. (NJCCCS 4.2, 4.5)

Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus. (NJCCCS 4.1, 4.5)

Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems. (NJCCCS 4.5)

Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral. (NJCCCS 4.3)

Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions. (NJCCCS 4.5)

Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement. (NJCCCS 4.1, 4.2, 4.5)

Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment. (NJCCCS 4.5)

Evaluation / Assessment Of Students

Student progress will be evaluated through class work, board work, homework assignments, and graded tests. The student's performance on the AP Exam or a college placement exam will also serve as methods of assessment.

Evaluation / Assessment Of Curriculum

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule(see attached).

Resources / Bibliography

College Board AP Central (www.apcentral.collegeboard.com)

New Jersey Core Curriculum Content Standards (Oct. 2004)
Calculus, A New Horizon 6th Edition, Howard Anton, John Wiley & Sons Inc.
1999.
Princeton Review AP Calculus Exam Workbook