BASIC GEOMETRY
2008

COURSE DESCRIPTION:

This course is designed for students to acquire a general knowledge of geometry. Topics addressed
by this course include Tools of Geometry, Basic Geometric Figures, Deductive Reasoning,
Transformations, Triangle Relationships, Measuring in Planes, and Space, Parallel Lines, Triangle, Congruence, Quadrilaterals, Similarity and Right Angle Trigonometry.
The course will be a combination of hands-on approach and a more formal, proof-centered approach. The hands-on approach will provide the students the opportunity to work together within lessons/activities with emphasis on discovery of concepts through concrete information.
Meanwhile, the formal, proof centered approach will emphasize deductive reasoning. Students will be able to understand how geometry relates to the real world and how often math is used in their lives today and in the future.

CORE CURRICULUM CONTENT STANDARDS:

STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE, AND ANALYZE PHENOMENA.
STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.
STANDARD 8.1 -TECHNOLOGY All students will use computer applications to gather and organize information and to solve problems. Technology, any modification of the natural world designed by human beings to solve human problems, enhance human life, or extend human capability, was identified by the United States Department of Labor as an essential workplace competency in a 1992 report called the Secretary's Commission on Achieving Necessary Skills (SCANS). SCANS stated that students should be able to select equipment and tools, apply technology to specific tasks, and maintain and troubleshoot equipment. The Department of Education recognized its importance by including technology in the original cross-content workplace readiness standards. In keeping with today's technological society, technological literacy has been further emphasized by its inclusion as a separate standards area which focuses on both computer and information literacy and technology education. Technology is evolving at a tremendous rate. Its ability to enhance human life, or extend human capability, was identified by the United States Department of Labor as an essential workplace

competency in a 1992 report called the Secretary's Commission on Achieving Necessary Skills (SCANS). SCANS stated that students should be able to select equipment and tools, apply technology to specific tasks, and maintain and troubleshoot equipment. The Department of Education recognized its importance by including technology in the original cross-content workplace readiness standards. In keeping with today's technological society, technological literacy has been further emphasized by its inclusion as a separate standards area which focuses on both computer and information literacy and technology education. Technology is evolving at an amazing rate, with both frequent advancements of existing technology and the creation of new technologies. All students must understand and be comfortable with the concepts and application of technology, not only in order to function in today's complex society, but also to become informed and productive adults of tomorrow. **Computer and Information Literacy** Computer and information literacy, which supports skills in information-gathering, information-organizing, and problem solving, has become critical for every student whether college- or workplace-bound. Colleges and employers are now demanding that students and employees possess a broad range of computer and information literacy proficiencies. More and more retail purchasing is being done on-line every year, and all but the most menial of positions now require a significant understanding of computer and information literacy. To ensure that students are computer literate, a separate standard that defines rigorous, in-depth learning has been included. The computer and information literacy standard is designed to be integrated and applied in all of the content areas of the Core Curriculum Content Standards. **Technology Education** The technology education standard was developed to ensure the literacy needed by all students to succeed in a highly technological world. Business and industry has clearly stated the need for technological skills in the workplace of the 21st Century. This standard is based on the Standards for Technological Literacy (STL): Content for the Study of Technology (ITEA, 2000), developed as part of the National Science Foundation (NSF)/National Aeronautics and Space Administration (NASA) funded by theTechnology for All Americans (TfAA) project. A study by DeKlerk has found that students form negative attitudes about the technological world if there are no formal technological experiences during the early school years. This finding Technological Literacy 1 Technological Literacy is a great concern to New Jersey business and industry. Other cognitive research suggests that "design-based learning" is important. Early studies with design and technology curriculum indicate that students who learn important technological concepts develop positive attitudes about technology, math, science and learning in general. For these reasons, an introduction to technology education, including engineering and technological design, is an essential component of a thorough and efficient K-12 education.

CUMULATIVE PROGRESS INDICATORS:

STANDARD 4.2 – MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12,** students will :

A. **Geometric Properties**

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway, hallway, or whether a ladder can reach a certain height on a building).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.

a. Investigate parallel lines . transversal, alternate interior angles, corresponding angles
b. Study triangles and their :

(1) Conditions for congruence and what occurs when there is a
(2) Segment joining midpoints of two sides is parallel to and half the length of the third side
(3) Describe the triangle Inequality and determine the

a. Minimal conditions for a shape to be a special quadrilateral and investigate
b. Circles, arcs, central and inscribed angles, chords, tangents
c. Calculate segments using self-similarity

4. Use reasoning and some form of proof to verify or refute conjectures and theorems.

a. Verification or refutation of proposed proofs
b. Simple proofs involving congruent triangles
c. Counterexamples to confirm conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.
3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.

a. Fractals (e.g., Sierpinski.s Triangle)
b. Patterns in areas and perimeters of self-similar figures
c. Outcome of extending iterative process indefinitely

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.

a. Distance between two points
b. Midpoint and slope of a line segment
c. Finding the intersection of two lines
d. Lines with the same slope are parallel
e. Lines that are perpendicular have slopes whose product is -1

2. Show position and represent motion in the coordinate plane using vectors

a. Addition and subtraction of vectors
b. Vector multiplication

D. Units of Measurement

1. Understand and use the concept of significant digits.

2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
3. Determine the degree of accuracy of a given measurement tool
4. Find the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.
a. Similar triangles
b. Pythagorean theorem
c. Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.
a. Approximation of area using grids of different sizes
b. Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
c. Estimation of area, perimeter, volume, and surface area

STANDARD 4.5 – MATHEMATICS

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
a) Open-ended problems
b) Non-routine problems
c) Problems with multiple solutions
d) Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., try a simpler problem. or make a diagram.) to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
a. Reading and writing
b. Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.

4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
a) Counterexamples as a means of disproving conjectures
b) Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
a. Concrete representations (e.g., base-ten blocks or algebra tiles)
b. Pictorial representations (e.g., diagrams, charts, or tables)
c. Symbolic representations (e.g., a formula)
d. Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences.

8.1 Technology

1. Use technology to gather, analyze, and communicate mathematical information. Use the geometry sketch pad and smart board software.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs. Use the geometry sketch pad and smart board software.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.

6. Use computer-based laboratory technology for mathematical applications in the sciences. Use the geometry sketch pad and smart board software.

Cumulative Progress Indicators (by grade 12):

Standard 1:

1.1 Demonstrate employability skills and work habits, such as work ethic, dependability, promptness, and getting along with others, needed to get and keep a job.
1.2 Describe the importance of personal skills and attitudes to job success.
1.3 Identify career interests, abilities, and skills.
1.4 Develop an individual career plan.
1.5 Identify skills that are transferable from one occupation to another.
1.6 Select a career major and appropriate accompanying courses.
1.7 Describe the importance of academic and occupational skills to achievement in the work world.
1.8 Demonstrate occupational skills developed through structured learning experiences, such as volunteer, community service, and work-based experiences or part-time employment.
1.9 Identify job openings.
1.10 Prepare a resume and complete job applications.
1.11 Demonstrate skills and attitudes necessary for a successful job interview.
1.12 Demonstrate consumer and other financial skills.

Standard 2:

2.1 Understand how technological systems function.
2.2 Select appropriate tools and technology for specific activities.
2.3 Demonstrate skills needed to effectively access and use technology-based materials through keyboarding, troubleshooting, and retrieving and managing information.
2.4 Develop, search, and manipulate databases.
2.5 Access technology-based communication and information systems.
2.6 Access and assess information on specific topics using both technological (e.g., computer, telephone, satellite) and print resources available in libraries or media centers.
2.7 Use technology and other tools to solve problems, collect data, and make decisions.
2.8 Use technology and other tools, including word-processing, spreadsheet and presentation programs, and print or graphic utilities, to produce products.
2.9 Use technology to present designs and results of investigations.
2.10 Discuss problems related to the increasing use of technologies.

Standard 3:

3.1 Recognize and define a problem, or clarify decisions to be made.
3.2 Use models, relationships, and observations to clarify problems and potential solutions.
3.3 Formulate questions and hypotheses.
3.4 Identify and access resources, sources of information, and services in the school and the community.
3.5 Use the library media center as a critical resource for inquiry and assessment of print and non-print materials.
3.6 Plan experiments.
3.7 Conduct systematic observations.

3.8 Organize, synthesize, and evaluate information for appropriateness and completeness.
3.9 Identify patterns and investigate relationships.
3.10 Monitor and validate their own thinking.
3.11 Identify and evaluate the validity of alternative solutions.
3.12 Interpret and analyze data to draw conclusions.
3.13 Select and apply appropriate solutions to problem-solving and decision-making situations.
3.14 Evaluate the effectiveness of various solutions.
3.15 Apply problem-solving skills to original and creative/design projects.

Standard 4:

4.1 Set short and long term goals.
4.2 Work cooperatively with others to accomplish a task.
4.3 Evaluate their own actions and accomplishments.
4.4 Describe constructive responses to criticism.
4.5 Provide constructive criticism to others.
4.6 Describe actions that demonstrate respect for people of different races, ages, religions, ethnicity and gender.
4.7 Describe the roles people play in groups.
4.8 Demonstrate refusal skills.
4.9 Use time efficiently and effectively.
4.10 Apply study skills to expand their own knowledge and skills.
4.11 Describe how ability, effort, and achievement are interrelated.

Standard 5:

5.1 Explain how common injuries can be prevented.
5.2 Develop and evaluate an injury prevention program.
5.3 Demonstrate principles of safe physical movement.
5.4 Demonstrate safe use of tools and equipment.
5.5 Identify and demonstrate the use of recommended safety and protective devices.
5.6 Identify common hazards and describe methods to correct them.
5.7 Identify and follow safety procedures for laboratory and other hands-on experiences.
5.8 Discuss rules and laws designed to promote safety and health, and their rationale.
5.9 Describe and demonstrate procedures for basic first aid and safety precautions

SUGGESTED ACTIVITES THAT ADDRESS THESE STANDARDS:

Standard 7 — Geometry and Spatial Sense — Grades 11-12

Indicators and Activities

The cumulative progress indicators for grade 12 appear below in boldface type. Each indicator is
followed by activities which illustrate how it can be addressed in the classroom in grades 11 and 12.

4.2.A.1 . Investigate, explore, and describe geometry in nature and real-world applications,
using *models, manipulative, and appropriate technology.

• Students use a computer-aided design (CAD) program to investigate rotations of objects in three dimensions.

• Students use *The Geometric SuperSupposer* to measure components of shapes and make observations. For example, they might construct parallelograms and measure sides,

angles, and diagonals, observing that opposite sides are congruent, as are opposite angles, and that diagonals bisect each other.

• Students use *The Geometer's Sketchpad* to investigate the effects of rotating a triangle about a fixed point.

• Students use commercial materials such as GeoShapes or Polydrons to construct three-dimensional geometric figures. They make tables concerning the number of vertices, edges, and faces in each solid. They measure the figures to determine their surface areas and volumes. They lay the patterns out flat to examine the nets of each solid. [A net is a flat shape which when folded along indicated lines will produce a three-dimensional object; for example, six identical squares joined in the shape of a cross can be folded to form a cube. Tabs added to the net facilitate attaching appropriate edges so that the shape remains three-dimensional.]

• Students work through the *Ice Cones* lesson that is described in the First Four Standards of this *Framework*. Students create a variety of paper cones out of circles with radius 10 inches, which are cut along a radius. They use graphing calculators to find the maximum volume of such cones.

• Students copy geometric designs using compass and straightedge, and generate their own designs.

• Students investigate wallpaper patterns, classifying them according to the transformations used. They study the structure of crystals from a geometric perspective.

4.2.A.1. Solve real-world and mathematical problems using geometric models.

• Students visit a construction site where the "framing" step of a building process is taking place. They note where congruence occurs (such as in the beams of the roof, where angles must be congruent). They write about why congruence is essential to buildings and other structures.

• Students use paper fasteners and tagboard strips with a hole punched near each end to investigate the rigidity of various polygon shapes. For shapes that are not rigid, they determine how to make the shape more rigid.

• Students draw a geometric representation and develop a formula to solve the problem of how many handshakes will take place if there are *n* people and each person shakes hands with each other person exactly once.

• Students work through the *On the Boardwalk* lesson that is described in the Introduction to this *Framework*. They determined the probability of winning a prize when tossing a coin onto a grid by having the coin avoid all of the grid lines.

• Students use graph models to represent a situation in which a large company wishes to install a pneumatic tube system that would enable small items to be sent between any of ten locales, possibly by relay. Given the cost associated with possible tubes (edges), the students work in groups to determine optimal pneumatic tube systems for the company. They report their results in letters written individually to the company president.

• Students work through the *Making Rectangles* lesson that is described in the First Four Standards of this *Framework*. They use combinations of algebra tiles which they try to arrange into rectangle shapes to help them develop procedures for multiplying binomials and factoring polynomials.

4.2.A.2. Analyze properties of three-dimensional shapes by constructing models and by drawing and interpreting two-dimensional representations of them.

• Pairs of students work together to describe and draw geometric figures. One student is given a picture involving one or more geometric figures and must describe the drawing to the other student without using the names of the figures. The second student, without seeing the figure, must visualize and represent the picture.
• Students create wind-up poster board models of rotational three-dimensional solids. They cut out a plane figure such as a circle or a rectangle from poster board, punch two holes in it near its edges, thread a cut rubber band through the holes, and attach the ends of the rubber band to the ends of a coat hanger from which the horizontal wire has been removed.
• They then twist the rubber band to wind up the figure and release to "show" the solid.
• Students use isometric dot paper to sketch figures made up of cubes. They also sketch top, front, and side views (projections) of the figure.
• One long-term project that some high school teachers use for assessment is the following:
  o Using a variety of means and materials, students begin by constructing models of the Platonic solids and other three dimensional geometric figures. They are then challenged to work in teams to find a relationship among the number of faces, vertices, and edges that holds for all of the solids (Euler's Formula: F + V ! E = 2).
• Students read or are shown the movie production of *Flatland: A Romance of Many Dimensions* by Edwin Abbott, a fascinating and imaginative story about life in a two-dimensional world.
• Students use a computer-aided design (CAD) program to investigate rotations of objects in three dimensions.

4.2.A.3. Understand and apply properties involving angles, parallel lines, and perpendicular lines.

• Students make tessellations with an assortment of different triangles, noting the variety of geometric patterns that are formed, including parallel lines, congruent angles, congruent triangles, similar triangles, parallelograms, and trapezoids.
• Students identify congruent angles on a parallelogram grid, and use their results to develop conjectures about alternate interior angles, corresponding angles of parallel lines, and opposite angles of a parallelogram.
• Working together, students review geometric vocabulary by sorting words written on index cards into groups and explaining their reasons for creating the groups they did. For example, they might place "parallelogram," "rhombus," "square," and "rectangle" in one group (since they are all parallelograms) and place "kite" and "trapezoid" in another group (since they are not parallelograms).
• Students find a variety of strategies for demonstrating that the sum of the measures of the angles of a triangle is 180E. Some use protractors and measure a pencil-and-paper figure, others create a triangle with *Geometer's Sketchpad* software and post the measures of the angles before dragging it from a vertex to notice that the sum always remains the same, and still others use a method that requires tearing each of the corners from an oak tag triangle and then fitting them together to make a line.

4.2.A.3. Use transformations, coordinates, and vectors to solve problems in Euclidean geometry.

• Students construct a polygon that outlines the top view of their school. They are asked to imagine that they are architects who need to send this outline by computer to a builder who has no graphics imaging capabilities. They develop strategies for sending this information to the builder. One group locates one corner of the building at the origin and determines the coordinates for the other vertices. Another group uses vectors to tell the builder what direction to proceed from the initial corner located at the origin.
• Students work on the question of where a power transformer should be located on a line so that the length of the cable needed to run to two points not on that line is minimized. They find that if the two points are on the same side of the line, then by using reflections they can construct a straight line that crosses the given line at the desired location.
• Students first determine the coordinates for the vertices of a parallelogram, a rhombus, a rectangle, an isosceles trapezoid, and a square with one vertex at the origin and a side along the x-axis. They then work in groups to determine where the coordinate system should be placed to simplify the coordinate selection for a kite, a rhombus, and a square.
• Students draw two congruent triangles anywhere in the plane and determine the minimum number of reflections needed to map one onto the other.
• Students draw a triangle on graph paper and then find the image of the triangle when the coordinates of each vertex are multiplied by various constants. They draw each resulting triangle and determine its area. They make a table of their results and look for relationships between the constants used for dilation and the ratios of the areas.
• Students use a Mira (Reflecta) to find the center of a circle, to draw the perpendicular bisectors of a line segment, or to draw the medians of a triangle.
• Students apply transformations to figures drawn on coordinate grids, record the coordinates of the original figure and its image, and look for patterns. They express these patterns verbally and symbolically. For example, flipping a point across the x-axis changes the sign of the y-coordinate so that the point (x,y) moves to (x, -y).
• Given the equation of a line, students plot the line on a coordinate grid and then shift the line according to a given translation. They then determine the equation of the resulting line. After doing several such problems, students identify patterns that they have found and write conjectures.

4.2.A.4. Use inductive and deductive reasoning to solve problems and to present reasonable explanations of and justifications for the solutions.

• In a computer-based, open-ended, assessment, groups of students use computer software to draw parallelograms, make measurements, and list as many properties of parallelograms and their diagonals as they can.
• Students prove deductively that a parallelogram is divided into two congruent triangles by a diagonal. They also prove that any angle inscribed in a semi-circle is a right angle. (An angle ABC is inscribed in a semi-circle if AC is a diameter and B is any other point on the circle.)
• Students explain in writing to a friend what the formula is for the measure of each interior angle in a regular polygon with *n* sides and how it is derived.
• Students build staircases from cubes, recording the number of steps and the total number of cubes used for each construction. They look for patterns, expressing them in words and symbolically in equations. They then try to justify their results using deductive reasoning.

• Students use *Cabri* software to investigate what happens when consecutive midpoints of a quadrilateral are connected in order. They state a conjecture based on their investigation and explain why they think it is true.
• Students investigate the relationship between the number of diagonals that can be drawn from one vertex of a polygon and the number of sides of that polygon. They write about their findings in their journals.
• Students work through the *A Sure Thing!?* lesson in the Introduction to this *Framewor*k.
They investigate the number of non-overlapping regions that can be created if they draw all the chords joining *n* points on the circumference of a circle.

4.2.B.4. Analyze patterns produced by processes of geometric change and express them in terms of iteration, approximation, limits, self-similarity, and fractals.

• Students duplicate the beginning stages of a fractal construction in the plane and analyze the sequences of their perimeters and their areas.
• Students use the reduction and enlargement capabilities of a copy machine to investigate the effects on area. They make a table showing the linear rate of reduction/enlargement and the resulting area for each successive reduction/enlargement. Then they graph the results — an
exponential function showing either decay or growth.
• Students use the slides and appropriate activities from *Fractals for the Classroom, Vol. 1* to analyze patterns produced by changes in geometric shapes.
• Students model decay in a bacterial culture by cutting a sheet of grid paper in half repeatedly and recording the area of each rectangle in a table. They then graph the number of cuts versus the area to see an example of exponential decay.
• Students plot the relationship between body height and arm length for people from one year of age through adulthood on coordinate grid paper and on log-log paper. They see that the graph is not a straight line on the coordinate grid paper; it is actually a logarithmic function. They find that the function appears as a straight line on log-log paper.

4.2.E.1. Use basic trigonometric ratios to solve problems involving indirect measurement.

• Students use trigonometric ratios to determine distances which cannot be measured directly, such as the distance between two points on opposite sides of a chasm.
• Students investigate how the paths of tunnels are determined so that people digging from each end wind up in the same place.
• Students use trigonometry to determine the cloud ceiling at night by directing a light (kept in a narrow beam by a parabolic reflector) toward the clouds. An observer at a specified distance measures the angle of elevation to the point at which the light is reflected from the cloud.
• Students plot the average high temperature for each month over the course of five years to see an example of a periodic function. They discuss what types of functions might be appropriate to represent this relationship.

4.2.E.2 . Develop, understand, and apply a variety of strategies for determining perimeter, area, *surface area, angle measure, and volume.

• Students find volumes of objects formed by combining geometric figures and develop formulas describing what they have done. For example, they might generate a formula for

finding the volume of a silo composed of a cylinder of specified radius and height topped by a hemisphere of the same radius.
• Students construct models to show how the volume of a pyramid with a square base and height equal to a side of the base is related to the volume of a cube with the same base.
• Students develop and use a spreadsheet to determine what the dimensions should be for a cylinder with a fixed volume, in order to minimize the surface area. Similarly, they investigate what should be the dimensions for a rectangle having a fixed perimeter in order to maximize the enclosed area. They discuss how the symmetry of these figures relates to the solutions.

4.2.E.2. Explore applications of other geometries in real-world contexts.

• Students represent lines using string and pins on Styrofoam balls (spheres). They analyze the properties of lines (e.g., all lines intersect) and triangles (e.g., it is possible to have a triangle with three 90E angles). They apply their results to finding the shortest route between two points on the earth.
• Students investigate the angel and devil drawings of M. C. Escher as examples of geometries in which there may be many "lines" through a given point that do not intersect a given "line." In this case, a "line" is an arc of a circle that is perpendicular to the outside circle of the drawing.
• Students explore another geometry using *Non-Euclidean Adventures on the Lénárt Sphere*.
• Students determine how many people are needed on a committee if there are to be four subcommittees, with each person on two subcommittees and each pair of subcommittees having one person in common. Most groups use letters to represent the individuals, and represent the four subcommittees by collections of letters, as in the following proposed solution {ABC, ADE, BDF, CEF}. The teacher asks the students to make a diagram of their solution, using "points" for people and "lines" for subcommittees, so that each subcommittee is a line whose points are its members. The rules for subcommittees become axioms about these points and lines; for example, "each person is on two subcommittees" becomes "each point is on two lines." The resulting geometry is an example of a finite geometry.

UNIT OBJECTIVES

Unit – Tools of Geometry - Basic Geometric Figures:

The students will be able to:

• Understand basic terms of geometry:
  o Point, space, line, collinear, plane, coplanar, postulate
• Become familiar with the tools needed to be successful in geometry.
• Utilize patterns and inductive reasoning
• Use inductive reasoning to make conjectures
• Understand basic postulates of geometry:
• Relate segments and rays to lines.
• Recognize parallel lines and parallel planes.
• Understand that many geometric figures, use only the parts of the lines called segments and rays.
• Find the lengths of segments.
• Measure angles with proficiency.

• Understand postulates of measuring (Ruler Postulate).
• Understand postulate of Segment Addition.
• Understand postulate of Angle Addition.
• Understand and measure angles and segments.
• Understand postulate of Protractor.
• Identify and understand properties of good definitions.
• Understand the meaning of midpoint, perpendicular bisector and angle bisector.
• Apply the Midpoint Formula
• Apply the Distance Formula
• Apply the definition of Slope
• Construct congruent segments and congruent angles.
• Utilize a straight edge and a compass to construct geometrics figures.
• Understand and utilize Constructions in relation to congruent segments and congruent angles.
• Understand the need for good definitions to help identify objects.
• Gain an understanding of deductive reasoning and be able to use it to solve problems and develop conjectures in math and real life.
• Understand the difference between a proof and a theorem.

INSTRUCTIONAL STRATEGIES:

Tools of Geometry - Basic Geometric Figures

Traditional Strategies:
• Lecture
• Black Board, white board &/or smart board work
• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:
• Students use paper folding to illustrate some geometrics terms such as: midpoint, angle bisector and perpendicular bisectors.
• Create a geometric shape in Origami;
  o Include a model
  o Instructions for making them
  o Geometric patterns within the design.
• Use of rubrics for projects.
• Cooperative Learning Activities
• Read a map and determine how long it will take to travel from one spot to another.

EVALUATION/ASSESSMENT OF STUDENTS:

Tools of Geometry - Basic Geometric Figures

• Teacher generated tests and quizzes
  o Multiple Choice Questions
  o Open-ended Questions
  o Writing Exercises
  o Quantitative Comparison Questions
• Book generated test and quizzes
• Homework
• Seat Work

• Class Participation
• Alternative Assessments
  o Listed above
  o Writing Assignment:
♣ Example:
• If I were a geometric shape what would I be? Why? And What Shapes would my cousins be? My mother and Father?

UNIT OBJECTIVES

Unit – Investigating Geometric Figures:

The students will be able to:

• Measure angles of a triangle.
• Classify triangles.
• Relate knowledge of triangles to real life situations such as design of furniture, buildings and bridges.
• Understand the Exterior Angle Theorem.
• Classify polygons.
• Understand the Polygon Exterior Angle-Sum Theorem.
• Graph lines in a coordinate plane.
• Recognize parallel and perpendicular lines by their slope.
• Classify and define quadrilaterals.
• Measure arcs of a circle and central angles.
• Create a circle graph.
• Understand postulates of Arc Addition.
• Utilize properties of congruence and similarity.
• Measure similar and congruent figures.

INSTRUCTIONAL STRATEGIES:

Investigating Geometric Figures

Traditional Strategies:

• Lecture
• Black or white &/or smart board work.
• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

• Students create a circle graph depicting how they spend their time in a given day.
• Use of rubrics for projects.
• Cooperative Learning Activities:
  o In a group setting build quadrilaterals using toothpicks, then draw and name each one.
• Have student identify parallel and perpendicular lines within a picture of a building or New York skyline.
• Graph the four slopes and identify them.
• Provide the students several drawings of figures containing polygons and have them identify them.

EVALUATION/ASSESSMENT OF STUDENTS:

Investigating Geometric Figures:

• Teacher generated tests and quizzes
  - Multiple Choice Questions
  - Open-ended Questions
  - Writing Exercises
  - Quantitative Comparison Questions
• Book generated test and quizzes
• Homework
• Seat Work
• Class Participation
• Alternative Assessments
  - Listed above
  - Writing Assignment:

♣ Example:

• Explain how angles and triangles are important in the design of furniture, buildings and bridges.
  - Provide a picture of at least two of the above each with the triangles highlighted.

UNIT OBJECTIVES

Unit – Transformations, Shapes in Motion

The students will be able to:

• Identifying isometrics.
• Locate reflection images of figures.
• Understand properties of Translation.
• Find translation images of figures.
• Utilize vectors and matrix addition to represent translations.
• Locate rotation images of figures.
• Identify real-life objects that involve rotation.
• Identify glide reflections.
• Understand how reflections are related to the other isometrics.
• Identify types of symmetry in figures.
• Understand how symmetry in figures influences art, dance, and tools for science.
• Locate dilation images of figures.
• Recognize applications of dilations in maps, photographs and other real life objects.

INSTRUCTIONAL STRATEGIES:

Transformations, Shapes in Motion

Traditional Strategies:

• Lecture
• Black , White and/or smart board work
• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

• Consider, Dilation in photography, explain enlargement and reduction.
• Use of rubrics for projects.
• Cooperative Learning Activities:
  - In a group research movies that the theme is based on dilation. Or
  - Explain how the showing of movies is related to dilation.

• Have students experience a kaleidoscope or a book with pictures showing the inside of one. Students discuss reflections of both.
• Graph the four slopes and identify them.

EVALUATION/ASSESSMENT OF STUDENTS:

Transformations, Shapes in Motion

• Teacher generated tests and quizzes
  o Multiple Choice Questions
  o Open-ended Questions
  o Writing Exercises
  o Quantitative Comparison Questions
• Book generated test and quizzes
• Homework
• Seat Work
• Class Participation
• Alternative Assessments
  o Listed above
  o Writing Assignment:

♣ Example:

Explain how the rotation of the moon around the earth is related to your understanding of rotation in mathematics.

UNIT OBJECTIVES

Unit – Triangle Relationships

The students will be able to:

• Analyze situations and become better problem solvers through use of logical reasoning.
• Use and apply properties of isosceles triangle.
• Write arguments using different styles of proofs.
• Utilize indirect reasoning to solve problems and relate it to real life situations.
• Use properties of mid-segments to solve problems.
• Identify coordinate proof.
• Understand properties of angle bisectors and perpendicular bisectors.
• Understand the Angle Bisector Theorem.
• Understand the Converse of Angle Bisector Theorem.

INSTRUCTIONAL STRATEGIES:

Triangle Relationships

Traditional Strategies:

• Lecture
• Black and white and/or smart board work.
• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

• Make a journal of types of triangles and properties of each.
• Use of rubrics for projects.
• Cooperative Learning Activities:
  o In a group create situations in real life that makes use of indirect reasoning.
  o IN a group students design and create kits that can fly.

♣ They must be able to describe the geometric properties of their design.
♣ A set of instructions of how to build such a kite must be included.

EVALUATION/ASSESSMENT OF STUDENTS:

Triangle Relationships

- Teacher generated tests and quizzes
  - Multiple Choice Questions
  - Open-ended Questions
  - Writing Exercises
  - Quantitative Comparison Questions
- Book generated test and quizzes
- Homework
- Seat Work
- Class Participation
- Alternative Assessments
  - Listed above
  - Writing Assignment:

♣ Example:

Explain, "The truth value of a converse does not depend on the truth value of the original statement."

UNIT OBJECTIVES

Unit – Measuring in the Plane

The students will be able to:

- Find area and perimeter of squares and rectangles.
- Use the process in finding the area to relate to real life situations,
- Find area of parallelograms and triangles.
- Understand and use the Pythagorean Theorem
- Understand and utilize the properties of 45 degree, 45 degree and 90 degree, and 30 degree, 60 degree and 90 degree triangles.
- Find the areas of trapezoids.
- Find the circumference of a circle.
- Find the length of an arc.
- Compute the area of circles, sectors and segments of a circle.

INSTRUCTIONAL STRATEGIES:

Measuring in the Plane

Traditional Strategies:

- Lecture
- Black , white and/or smart board work
- Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

- Make a journal of types of triangles and properties of each.
- Use of rubrics for projects.
- Cooperative Learning Activities:
  - The class can be divided into several small groups to work on a class quilt.

♣ Each group must identify the geometric shapes within their section of the

quilt.
- Students, in groups will design gardens for a landscaper.

♣ Each group must relate geometrics shapes and the use of area to design such paths, walls or gardens.

EVALUATION/ASSESSEMENT OF STUDENTS:

Triangle Relationships

- Teacher generated tests and quizzes
  - Multiple Choice Questions
  - Open-ended Questions
  - Writing Exercises
  - Quantitative Comparison Questions
- Book generated test and quizzes
- Homework
- Seat Work
- Class Participation
- Alternative Assessments
  - Listed above
  - Writing Assignment:

♣ Example:

Explain, how the formula for the area of a trapezoid is related to the formula for the area of a triangle.

UNIT OBJECTIVES

Unit – Measuring in Space

The students will be able to:

- Recognize nets of various space figures.
- Use surface areas of objects in daily life.
- Find the lateral area and surface of areas of pyramids and cones.
- Find the volumes of prisms and cylinders.
- Use finding volume in real life situations.
- Find volumes of pyramids and cones.
- Solve problems concerning the amount of space in things.
- Use geometric models to find the probability of events.

INSTRUCTIONAL STRATEGIES:

Measuring in Space

Traditional Strategies:

- Lecture
  0 Black, white and/or smart board work
- Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

- Make a journal of types of triangles and properties of each.
- Use of rubrics for projects.
- Cooperative Learning Activities:
  - In a group have students identify how geometric probability to represent occurrences of events.

- Have students work together to divide the figure and to find the dimensions of each piece.

EVALUATION/ASSESSMENT OF STUDENTS:

Measuring in Space

- Teacher generated tests and quizzes
  - Multiple Choice Questions
  - Open-ended Questions
  - Writing Exercises
  - Quantitative Comparison Questions
- Book generated test and quizzes
- Homework
- Seat Work
- Class Participation
- Alternative Assessments
  - Listed above
  - Writing Assignment:

♣ Example:

Explain, how someone could confuse odds with probability.

UNIT OBJECTIVES

Unit – Reasoning and Parallel Lines

The students will be able to:

- Identify pairs of angles formed by two lines and a transversal.
- Relate the measure of angles formed by parallel lines and a transversal.
- Understand how parallel lines are used in building and construction.
- Recognize conditions that result in parallel lines..
- Write proofs that involve parallel lines..
- Understand how parallel lines are used in real life situations such as graphic arts.
- Understand the Converse of Alternate Interior Angles Theorem.
- Understand the Converse of Same-Side Interior Angles Theorem.

INSTRUCTIONAL STRATEGIES:

Reasoning and Parallel Lines

Traditional Strategies:

- Lecture
- Black , white and /or smart board work
- Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

- Students make a frame for a picture of parallel lines representing a building.
- Use of rubrics for projects.
- Cooperative Learning Activities:
  - In groups have students identify parallel lines in a picture of a building or construction.

EVALUATION/ASSESSMENT OF STUDENTS:

Reasoning and Parallel Lines

- Teacher generated tests and quizzes

- Multiple Choice Questions
- Open-ended Questions
- Writing Exercises
- Quantitative Comparison Questions

• Book generated test and quizzes
• Homework
• Seat Work
• Class Participation
• Alternative Assessments
- Listed above
- Writing Assignment:

♣ Example:
Brainstorm letters that may show related angles.

UNIT OBJECTIVES
Unit – Proving Triangles Congruent
The students will be able to:
• Prove two triangles are congruent using the SSS and SAS postulates.
• Prove two triangles are congruent using the ASA and AAS postulates.
• Prove two triangles are congruent using the HL postulate.
• Use CPCTC to prove that parts of two triangles are congruent.
• Identify congruent overlapping triangles.
• Prove two triangles are congruent by first proving two other triangles are congruent.

INSTRUCTIONAL STRATEGIES:
Proving Triangles Congruent
Traditional Strategies:
• Lecture
• Black, white &/or smart board work
• Constructions
• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:
• Cooperative Learning Activities:
- Students brainstorm to list examples of congruent triangles in real life..

EVALUATION/ASSESSMENT OF STUDENTS:
Proving Triangles Congruent
• Teacher generated tests and quizzes
- Multiple Choice Questions
- Open-ended Questions
- Writing Exercises
- Quantitative Comparison Questions

• Book generated test and quizzes
• Homework
• Seat Work
• Class Participation
• Alternative Assessments

- Listed above
- Writing Assignment:

♣ Example: Formal proof on Base Angles of an Isosceles Triangle.

UNIT OBJECTIVES

Unit – Quadrilaterals

The students will be able to:

- Find relationships between angles, sides and diagonals of parallelograms.
- Prove that a quadrilateral is a parallelogram.
- List the similarities and differences between rectangles, squares and rhombuses.
- List the properties of trapezoids and kites
- Locate quadrilaterals on coordinate planes

INSTRUCTIONAL STRATEGIES:

Quadrilaterals

Traditional Strategies:

- Lecture
- Black white and/or smart board work
- Constructions
- Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

- Cooperative Learning Activities:
  - Students find real life examples of quadrilaterals.

EVALUATION/ASSESSMENT OF STUDENTS:

Quadrilaterals

- Teacher generated tests and quizzes
  - Multiple Choice Questions
  - Open-ended Questions
  - Writing Exercises
  - Quantitative Comparison Questions
- Book generated test and quizzes
- Homework
- Seat Work
- Class Participation
- Alternative Assessments
  - Listed above
  - Writing Assignment:

♣ Example: Formal proof of Diagonals of an Isosceles Trapezoid are Congruent.

UNIT OBJECTIVES

Unit – Similarity

The students will be able to:

- Find relationship between ratio, proportion and similar triangles.
- Prove two triangles are similar using the AA Similarity postulate.
- Prove two triangles are similar using the SAS Similarity theorem.
- Prove two triangles are similar using the SSS Similarity theorem.
- Demonstrate the relationship between a right triangle and two triangles formed by

constructing an altitude to the hypotenuse
• Find proportional relationships between similar triangles

INSTRUCTIONAL STRATEGIES:

Similarity

Traditional Strategies:
• Lecture
• Black, white and/or smart board work
• Constructions
• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:
• Cooperative Learning Activities:
o Students create enlarged replicas of candy bar (wrappers) for an advertising application.

EVALUATION/ASSESSMENT OF STUDENTS:

Similarity

• Teacher generated tests and quizzes
o Multiple Choice Questions
o Open-ended Questions
o Writing Exercises
o Quantitative Comparison Questions
• Book generated test and quizzes
• Homework
• Seat Work
• Class Participation
• Alternative Assessments
o Listed above
o Writing Assignment:
♣ Example: Students use their understanding of similar triangles to find missing lengths.

UNIT OBJECTIVES

Unit – Right Angle Trigonometry

The students will be able to:

• Calculate the tangents of the acute angles of a right triangle.
• Use tangents to determine the side lengths in right triangles.
• Calculate the value of the sines and cosines of the acute angles of a right triangle.
• Use sine and cosine to determine the unknown side lengths in right triangles.
• Identify the angles of elevation and depression
• Use the angles of elevation and depression and trigonometric ratios to solve problems

INSTRUCTIONAL STRATEGIES:

Right Angle Trigonometry

Traditional Strategies:
• Lecture
• Black, white and/or smart board work
• Constructions

• Use of Open-ended problems, written exercises and quantitative comparison activities.

Alternative Assessment:

• Cooperative Learning Activities:

o Students in groups find the height of a telephone pole or the peak of a roof.

EVALUATION/ASSESSMENT OF STUDENTS:

Right Angle Trigonometry

• Teacher generated tests and quizzes

o Multiple Choice Questions

o Open-ended Questions

o Writing Exercises

o Quantitative Comparison Questions

• Book generated test and quizzes

• Homework

• Seat Work

• Class Participation

• Alternative Assessments

o Listed above

o Writing Assignment:

♣ Example: Students use their understanding of trigonometry to state how to find the distance across a wide river.

EVALUATION/ASSESSMENT OF CURRICULUM:

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule (see attached).

RESOURCES/BIBLIOGRAPHY:

"Hopatcong High School Basic Geometry – Grades 10-12", July, 1997

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"New Jersey Mathematics Curriculum Framework", Joseph G. Rosenstein, Janet H. Caldwell, Warren D. Crown, December, 1996

Geometry – Tools for a Changing World, Laurie Bass, Basia Hall, Art Johnson and Dorothy Wood, Prentice Hall, 2002