

# College Mathematics

## 2008

### **COURSE DESCRIPTION:**

“To most outsiders, modern mathematics is unknown territory. Its borders are protected by dense thickets of technical terms; its landscapes are a mass of indecipherable equations and incomprehensible concepts. Few realize that the world of modern mathematics is rich with vivid images and provocative ideas.” *Ivars Peterson, The Mathematical Tourist*

The connection between the mathematics presented in this course and down-to-earth, concrete real-life problems is direct and immediate.

This semester course will begin with a unit covering the mathematics of social choice including voting systems, fair division schemes and apportionment methods applied historically in the United States and elsewhere. Mastery of these methods, which relate directly to the decision-making at all levels of government, will allow the student to identify and correct unfair practices encountered in the workplace and in government.

The second unit, on management science, deals with methods of solving real-world problems involving the organization and management of complex activities. Specifically, activities involving either a large number of steps and/or a large number of variables (routing the delivery of mail, placing a spaceship on mars, organizing a banquet, scheduling classrooms at a large institution, etc.).

In addition, this course will include a unit on growth that will study linear, geometric and logistic growth. The student will analyze symmetry within the growth methods as well as the Fractal Geometry found in science and nature.

The final unit covering statistics concentrates on the various statistical methods applied and the reliability of each. Students will explore the limited use of box plots and the extensive real-world use of the normal curve.

## CORE CURRICULUM CONTENT STANDARDS

### The Vision

The vision of the mathematics standards is focused on achieving one crucial goal: To enable ALL of New Jersey's children to acquire the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives. We want ALL students to achieve the standards. There may be exceptions, but those exceptions should be exceptional.

Perhaps the most compelling reason for this vision is that all of our children, as well as our state and our nation, will be better served by higher expectations, by curricula that go far beyond basic skills and include a variety of mathematical models, and by programs which devote a greater percentage of instructional time to problem-solving and active learning.

Many students respond to the traditional curriculum with boredom and discouragement. They feel that mathematics will never be useful in their lives, and they develop the perception that success in mathematics depends on some innate ability that they simply do not have. We must overcome the feelings among students that they don't like mathematics, they don't need mathematics, and they can't do mathematics. Curricula that evoke these responses in students, curricula that assume student failure, are bound to fail; we need to develop curricula that assume student success.

Our curricula are often preoccupied with what national reports call "shopkeeper arithmetic" competency in the basic operations that were needed to run a small store several generations ago. The economy in which graduates of our schools will seek employment is more competitive than ever and is rapidly changing in response to advances in technology. To compete in today's global, information-based economy, students must be able to solve real problems, reason effectively, and make logical connections.

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"Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents, and teachers all expect that most students can master mathematics if only they work hard enough. The record of accomplishment in these countries - and in some intervention programs in the United States - shows that most students can learn much more mathematics than is commonly assumed in this country."

Everybody Counts, Mathematical Sciences Education Board, National Academy of Sciences (1989)

**STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)**

ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

**Descriptive Statement:** Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

**Data Analysis (or Statistics).** In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

**Discrete Mathematics•Systematic Listing and Counting.** Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

**Discrete Mathematics•Vertex-Edge Graphs and Algorithms.**

Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms", and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in our society, and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data supported arguments.

**STANDARD 8.1:** All students will use computer applications to gather and organize information to solve problems.

**STANDARD 9.1:** All Students will develop career awareness and planning, employability skills, and foundational knowledge necessary for success in the workplace.

**STANDARD 9.2:** All students will demonstrate critical life skills in order to be functional members of society.

## Cumulative Progress Indicators:

**(4.4A) DATA ANALYSIS** *Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12**, students will:*

1. Use surveys and sampling techniques to generate data and draw conclusions about large groups. \*Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling)
2. Evaluate the use of data in real-world contexts. \*Accuracy and reasonableness of conclusions drawn, *Correlation vs. causation*, Bias in conclusions drawn (e.g., influence of how data is displayed), Statistical claims based on sampling,
3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.
5. Analyze data using technology, and use statistical terminology to describe conclusions.
  - Measures of dispersion: variance, standard deviation, outliers
  - Correlation coefficientNormal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)
6. Distinguish between randomized experiments and observational studies.

**(4.4D) Discrete Mathematics-Vertex-Edge Graphs and Algorithms** *-Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12**, students will:*

1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems.
  - Circuits that include every edge in a graph
  - Circuits that include every vertex in a graph
  - Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring
  - Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures)
2. Explore strategies for making fair decisions.
  - Combining individual preferences into a group decision (e.g., determining winner of an election or selection process)Determining how many Student Council representatives each class (9<sup>th</sup>, 10<sup>th</sup>, 11<sup>th</sup>, and 12<sup>th</sup> grade) gets when the classes have unequal sizes (apportionment)

**(8.1)**

**A Basic Computer Skills and Tools**

1. Use appropriate technology vocabulary.
2. Use common features of an operating system
3. Demonstrate effective input of text and data, using touch keyboarding with proper technique.
4. Input and access data and text efficiently and accurately through proficient use of other input devices, such as the mouse.
5. Create documents with advanced text-formats, graphics using word processing.
6. Create a file containing customized information by merging documents.
7. Construct a simple spreadsheet, enter data, and interpret the information.
8. Design and produce a basic multimedia project.
9. Plan and create a simple database, define fields, input data, and produce a report using sort and query.
10. Use network resources for storing and retrieving data.
11. Choose appropriate electronic graphic organizers to create, construct, or design a document.
12. Create, organize and manipulate shortcuts.

**B. Application of Productivity Tools**

**Social Aspects**

1. Demonstrate an understanding of how changes in technology impact the workplace and society.
2. Exhibit legal and ethical behaviors when using information and technology, and discuss consequences of misuse.
3. Explain the purpose of an Acceptable Use Policy and the consequences of inappropriate use of technology.
4. Describe and practice safe Internet usage.
5. Describe and practice “etiquette” when using the Internet and electronic mail.

**Information Access and Research**

6. Choose appropriate tools and information resources to support research and solve real world problems, including but not limited to:
  - On-line resources and databases
  - Search engines and subject directories
7. Evaluate the accuracy, relevance, and appropriateness of print and non-print electronic information sources.

**Problem Solving and Decision Making**

8. Use computer applications to modify information independently and/or collaboratively to solve problems.
9. Identify basic hardware problems and demonstrate the ability to solve common problems.
10. Determine when technology tools are appropriate to solve a problem and make a decision.

**(9.1)** Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

**A. Career Awareness/Preparation** 1. [Analyze] Re-evaluate personal interests, abilities, and skills through various measures including self assessments.

2. Evaluate academic and career skills needed in various career clusters.

3. Analyze factors that can impact an individual's career.

4. Review and update one's career plan and include in a portfolio.

5. Research current advances in technology that apply to a selected occupational career cluster.

**B. Employability Skills**

1. Assess personal qualities that are needed to obtain and retain a job related to career clusters.

2. Communicate and comprehend written and verbal thoughts, ideas, directions, and information relative to educational and occupational settings.

3. Select and utilize appropriate technology in the design and implementation of teacher approved projects relevant to occupations and/or higher educational settings.

4. Evaluate the following academic and career skills as they relate to home, school, community, and employment:

Communication

Punctuality

Time management

Organization

Decision making

Goal setting

Resources allocation

Fair and equitable competition

Safety

Employment application skills

Teamwork

5. Demonstrate teamwork and leadership skills that include student participation in real world applications of career and technical education skills [through student organizations].

**All students electing further study in career and technical education will also:**

6. Participate in a structured learning experience that demonstrates interpersonal communication, teamwork, and leadership skills.

7. Participate in simulated industry assessments when and where appropriate.

8. Prepare industry-specific technical reports/projects that incorporate graphic aids when and where appropriate.

9. Demonstrate occupational health and safety skills related to industry-specific activities.

**(9.2)** Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

**A. Critical Thinking**

1. Apply communications and data analysis to the problem-solving and decision making processes [to] in a variety of life situations.
2. Describe and apply constructive responses to criticism.
3. Apply the use of symbols, pictures, graphs, objects, and other visual information to a selected project in academic and/or occupational settings.
4. Recognize bias, vested interest, stereotyping and the manipulation and misuse of information while formulating solutions to problems that interfere with attaining goals.
5. Apply knowledge and skills needed to use various means of transportation within a community.

**B. Self-Management**

1. Revise and update the personal growth plan to address multiple life roles.
2. Apply project planning and management skills in academic and/or occupational settings.
3. Compare and contrast methods for maximizing personal productivity.

**C. Interpersonal Communication**

1. Model interpersonal and effective conflict resolution skills [to peers].
2. Communicate effectively in a variety of settings with a diverse group of people.

**D. Character Development and Ethics**

1. Analyze how character influences work performance.
2. Identify and research privileges and duties of citizens in a democratic society.
3. Discuss consequences and sanctions when on-the-job rules and laws are not followed.
4. Compare and contrast a professional code of ethics or code of conduct from various work fields and discuss similarities and differences.
5. Apply a professional code of ethics to a workplace problem or issue.

SUGGESTED ACTIVITIES THAT ADDRESS THESE STANDARDS MAY INCLUDE, BUT ARE NOT LIMITED TO:

**(4.4A)**

Relating to statistics

- Show sampling methods
- Discuss concerns with reporting methods
- Determine box plots for data description
- Define mean, first quartile, third quartile, range and median
- Define standard deviation

Relating to the mathematics of social choice

- Define preference ballots and preference schedules
- Define plurality method and majority rule
- Describe what is wrong with the plurality method
- Define the Condorcet criterion
- Demonstrate the Borda count method
- Describe what is wrong about the Borda count method
- Show the Plurality-with-Elimination method
- Describe what is wrong about the Plurality-with-Elimination method
- Illustrate the method of pairwise comparisons
- Show what is wrong with the method of pairwise comparisons
- Illustrate a method for ranking candidates
- Show how to rank candidates using the Recursive Plurality-with-Elimination method
- Describe Fairness criteria
- Show the Banzhof power distribution in weighted voting
- Show fair division schemes: divider-chooser, lone-divider, lone-chooser, last diminisher, and sealed bids
- Demonstrate the use of the method of markers
- Identify the apportionment problem
- Define and obtain a state's standard quota
- Illustrate Hamilton's method
- Describe the Alabama paradox
- Show the new state's paradox
- Describe Jefferson's method
- Show how to apply the quota rule
- Demonstrate Adam's method and Webster's method

Relating to the mathematics of growth

- Differentiate between linear, geometric and logistic growth
- Discuss and illustrate the Fibonacci sequence
- Illustrate how to find a Fibonacci number
- Demonstrate the mathematics of the golden ratio
- Define and determine gnomons and gnomonic growth
- Define and use the logistic growth equation

#### **(4.4D)**

- Define and illustrate a network graph
  - Define and illustrate vertices, edges, loops, multiple edges,
  - Define complete graphs
  - Demonstrate unicursal tracings and show how to discriminate between closed and open tracings of this type
  - Illustrate and define connected and disconnected graphs
  - Define adjacent vertices and determine the degree of a vertex
  - Define and illustrate paths and circuits
  - Define and illustrate Euler and Hamilton circuits
  - Show how to Eulerize a graph
  - Show how to find an Euler circuit in a connected graph using Fleury's Algorithm
  - Be able to explain why different looking graphs are the same graph
  - Determine the number of Hamilton circuits in a complete graph of N vertices
  - Define a weighted graph
  - Illustrate how to determine the weight of graphs that visit every edge
  - Illustrate how to determine the lightest circuit that visits every vertex
  - Define the Traveling Salesperson Problem
  - Demonstrate the brute force, nearest neighbor and cheapest link algorithms
  - Define subgraphs
  - Define a spanning subgraph
  - Define a tree
  - Identify and illustrate the four properties of a tree
  - Show Kruskal's algorithm for determining a minimum spanning tree for a given weighted graph
  - Determine Steiner points
  - Illustrate finding the shortest network linking three or four points
- Relating to the mathematics of scheduling
- Define processor, task, precedence relations and processing time
  - Define digraphs, cycles, indegree, outdegree and paths
  - Show how to use the decreasing time algorithm and the back flow algorithm
  - Demonstrate how to discover the critical path for a project
  - Define and compute relative percentage of error

#### **(8.1)**

Use a computer to research, prepare, and present individual or group projects relating to the above tasks.

## INSTRUCTIONAL STRATEGIES:

Included but not limited to, would be the following:

- Use of committee settings, randomly selected from the class for purposes of analyzing graphs, determining shortest routes, analyzing voting methods, drawing a tree of four vertices and determining the minimum spanning tree, etc.
- A small amount of lecture at the outset or conclusion of such group work
- Groups gathering data from schoolmates and/or school staff
- Using the software Geometry Sketchpad to draw graphs, determine spanning trees, etc.
- Making use of the power point capability of the classroom computer projector to display work done on the Sketchpad
- Groups will be using this capability also

## EVALUATION/ASSESSMENT OF STUDENTS:

Included but not limited to would be the following:

- Announced tests
- Announced and unannounced quizzes
- Projects
- Evaluations of group and individual presentations

## EVALUATION/ASSESSMENT OF CURRICULUM

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule.

## RESOURCES/BIBLIOGRAPHY:

- Finite Mathematics with Applications, Zitarelli and Coughlin, HBJ
- Introduction to Statistics and Data Analysis, Peck, Olsen, Devore, Duxsbury
- The Practice of Statistics, Yates, Moore, McCabe, Freeman
- Excursions in Modern Mathematics, Tannenbaum and Arnold, Prentiss Hall
- Understanding Basic Statistics, Brase & Brase, Houghton Mifflin
- Rutgers' Leadership Program in Discrete Mathematics
- The Mathematics Forum
- Eisenhower Consortium for Better Schools