

Pre-Calculus Honors

2009

COURSE DESCRIPTION: Pre-Calculus Honors introduces aspects of higher mathematics to college preparatory students who are preparing to take AP Calculus. It includes an extension of Algebra, Trigonometry, and analytical Geometry as well as selected topics such as functions (polynomial, rational, exponential, logarithmic, and trigonometric), number systems, limits, continuity, and a rigorous, graphing intensive introduction to differential Calculus.

CORE CURRICULUM STANDARDS:

STANDARD 4.1 (Number and numerical operations) All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

STANDARD 4.2 (Geometry and measurement) All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

STANDARD 4.3 (Patterns and algebra) All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes

STANDARD 4.4 (Data analysis, probability, and discrete mathematics) All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data

STANDARD 4.5 (Mathematical processes) All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas

STANDARD 8.1 (Computer and information literacy) All students will use computer applications to gather and organize information and to solve problems.

STANDARD 8.2 (Technology Education) All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual, society, and the environment.

CUMUATIVE PROGRESS INDICATORS:

STANDARD 4.1 (Number and numerical operations)

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.
3. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

STANDARD 4.2 (Geometry and measurement)

A. Geometric Properties

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway).
2. Apply the properties of geometric shapes.
 - ♣ Triangles
 - a. Triangle Inequality
 - ♣ Counterexamples to incorrect conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.
 - ♣ Distance between two points
 - ♣ Midpoint and slope of a line segment
 - ♣ Finding the intersection of two lines

- ♣ Lines with the same slope are parallel
- ♣ Lines that are perpendicular have slopes whose product is -1

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
 - ♣ Degree of accuracy of a given measurement tool
 - ♣ Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.
 - ♣ Similar triangles
 - ♣ Pythagorean theorem
 - ♣ Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.

- ♣ Approximation of area using grids of different sizes
- ♣ Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
- ♣ Estimation of area, perimeter, volume, and surface area

STANDARD 4.3 (Patterns and algebra)

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.
 - ♣ Explicit formulas for n^{th} terms
 - ♣ Sums of finite arithmetic series
 - ♣ Sums of finite and infinite geometric series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.

2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.

- ♣ Slope of a line or curve
- ♣ Domain and range
- ♣ Intercepts
- ♣ Continuity
- ♣ Maximum/minimum
- ♣ Estimating roots of equations
- ♣ Intersecting points as solutions of systems of equations
- ♣ Rates of change

3. Understand and perform transformations on commonly-used functions.

- ♣ Translations, reflections, dilations
- ♣ Effects on linear and quadratic graphs of parameter changes in equations
- ♣ Using graphing calculators or computers for more complex functions

4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.

- ♣ Linear vs. non-linear
- ♣ Symmetry
- ♣ Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.

- ♣ Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
- ♣ Direct and inverse variation
- ♣ Absolute value
- ♣ Expressions, equations and inequalities
- ♣ Same function can model variety of phenomena
- ♣ Growth/decay and change in the natural world
- ♣ Applications in mathematics, biology, and economics (including compound interest)

2. Analyze and describe how a change in an independent variable leads to change in a dependent one.
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Evaluate and simplify expressions.
 - ♣ Add and subtract polynomials
 - ♣ Multiply a polynomial by a monomial or binomial
 - ♣ Divide a polynomial by a monomial
2. Select and use appropriate methods to solve equations and inequalities.
 - ♣ Linear equations - algebraically
 - ♣ Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula
 - ♣ All types of equations using graphing, computer, and graphing calculator techniques
3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

STANDARD 4.4 (Data analysis, probability, and discrete mathematics)

A. Data Analysis (or Statistics)

1. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.
2. Analyze data using technology, and use statistical terminology to describe conclusions.
 - ♣ Correlation coefficient

B. Discrete Mathematics—Systematic Listing and Counting

1. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability.

STANDARD 4.5 (Mathematical processes)

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - ♣ Open-ended problems
 - ♣ Non-routine problems
 - ♣ Problems with multiple solutions
 - ♣ Problems that can be solved in several ways

3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
 - ♣ Reading and writing
 - ♣ Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).

3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
 - ♣ Counterexamples as a means of disproving conjectures
 - ♣ Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
 - ♣ Concrete representations (e.g., base-ten blocks or algebra tiles)
 - ♣ Pictorial representations (e.g., diagrams, charts, or tables)
 - ♣ Symbolic representations (e.g., a formula)
 - ♣ Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use graphing calculators and computer software to investigate properties of functions and their graphs.
3. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).

STANDARD 8.1 (Computer and information literacy)

Basic Computer Skills and Tools

1. Construct a spreadsheet, enter data, use mathematical or logical functions to manipulate and process data, generate charts and graphs, and interpret the results.
2. Given a database, define fields, input data from multiple records, produce a report using sort and query, and interpret the data.

B. Application of Productivity Tools

Social Aspects

1. Describe the potential and implications of contemporary and emerging computer applications for personal, social, lifelong learning, and workplace needs.
2. Exhibit legal and ethical behaviors when using information and technology, and discuss consequences of misuse.
3. Make informed choices among technology systems, resources, and services in a variety of contexts.
4. Use appropriate language when communicating with diverse audiences using computer and information literacy.

Information Access and Research

5. Select and use specialized databases for advanced research to solve real world problems.
6. Evaluate information sources for accuracy, relevance, and appropriateness.

Problem Solving and Decision Making

7. Create and manipulate information, independently and/or collaboratively, to solve problems and design and develop products.
8. Identify a problem in a content area and formulate a strategy to solve the problem using brainstorming, flowcharting, and appropriate resources.

STANDARD 8.2 (Technology Education)

Nature and Impact of Technology

1. Use appropriate data to discuss the full costs, benefits and trade-offs, and risks related to the use of technologies.
2. Explain how technological development is affected by competition through a variety of management activities associated with planning, organizing, and controlling the enterprise.
3. Provide various examples of how technological developments have shaped human history.

B. Design Process and Impact Assessment

1. Develop methods for creating possible solutions, modeling and testing solutions, and modifying proposed design in the solution of a technological problem using hands-on activities.
- 2.

SUGGESTED ACTIVITIES THAT ADDRESS THESE STANDARDS MAY INCLUDE BUT ARE NOT LIMITED TO:

STANDARD 4.1 (Number and numerical operations)

A. Number Sense

1. PHPC read page 866, p873 9-16, 21, 22

B. Numerical Operations

1. PHPC page 874 71-80
2. AA sections 7-1, 7-2, 7-3, 7-7, and 7-8

C. Estimation

1. PHPC pages 852-857, page 857-858 #1-26

STANDARD 4.2 (Geometry and measurement)

A. Geometric Properties

1. PHPC pages 875-878, page 878 #1-41

B. Transforming Shapes

1. PHPC pages 86-96, page 96 #1-74

C. Coordinate Geometry

1. PHPC pages 2-6, page 7 #1-56

D. Units of Measurement

1. PHPC pages 927-934, page 934 #1-28

E. Measuring Geometric Objects

1. PHPC pages 448-456, page 456 #63
2. AA sections 5-9 and 5-10

STANDARD 4.3 (Patterns and algebra)

A. Patterns

1. PHPC pages 742-750, page 150 #1-66
2. PHPC pages 163-170, page 170 #1-42
3. PHPC pages 769-772, #1-32

B. Functions and Relationships

1. PHPC pages 824-828, page 88 #1-42
2. AA sections 1-4, 2-5, 2-6, 3-2, 3-3, 5-2, 6-10
3. AA sections 4-4 through 4-10
4. PHPC pages 71 – 72, page 75 #1-18

C. Modeling

1. PHPC pages 80-81, page 84 #1-36
2. PHPC pages 232-241, page 242 #1-60
3. PHPC pages 349-359, page 359 #1-80
4. AA section 2-3
5. Nasco Joke Worksheets page 57 and 58

D. Procedures

1. PHPC pages 889-895, page 895 #1-48
2. Nasco Joke Worksheets pages 30-34

STANDARD 4.4 (Data analysis, probability, and discrete mathematics)

A. Data Analysis (or Statistics)

1. PHPC pages 37-40, page 40 #1-17
2. AA section 3-6

B. Discrete Mathematics—Systematic Listing and Counting

1. PHPC pages 752-756, page 756 #1-51

STANDARD 4.5 (Mathematical processes)

A. Problem Solving

Problem solving strategies should be taught throughout the course. Providing students with open-ended, non routine problems, problems with multiple solutions, and problems that can be solved in a variety of ways should be ongoing throughout the year, aligned with pre-calculus topics.

B. Communication

Communication is essential to the learning process. Students must be asked to verbally explain their thought processes on solving problems and be able to convey their ideas to their peers. This should be ongoing throughout the year especially when reviewing homework.

C. Connections

1. PHPC (teacher's edition) pages xxv-xxviii

E. Representations

A well organized and legible notebook is essential for success in this course.

Students should be encouraged to use a variety of examples in each lesson.

Pictorials, symbolic and graphical representations should be used.

F. Technology

Graphing calculators should be used to organize, display, and analyze as often as possible.

STANDARD 8.1 (Computer and information literacy)

Technology is evolving at an amazing rate. All students must understand and be comfortable with the concepts and application of technology in today's society. Computer and information literacy has become critical for every student whether college or workplace bound.

INSTRUCTIONAL STRATEGIES:

Instructional strategies used should include but not be limited to the following:

1. Lecture
2. Black, White, and or Smart Board work
3. TI 83/84 Graphing calculators
4. Use of open-ended problems, written and oral exercises
5. Group work
6. Cooperative learning
7. Vocabulary

EVALUATION/ASSESSMENT OF STUDENTS:

1. Regular and frequent quizzes comprised of a variety of problem types ranging from multiple choice to open-ended questions.
2. Periodic, comprehensive examinations comprised of a variety of problem ranging from multiple choice to short answer to open-ended.
3. Regular projects involving applications of mathematical concepts extensively utilizing technology and or graphing calculators.

EVALUATION/ASSESSMENT OF CURRICULUM:

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule (see attached).

RESOURCES/BIBLIOGRAPHY:

PHPC: Prentice Hall Pre-Calculus 6th edition (textbook)

**AA: Advanced Algebra University of Chicago School Mathematics Project
(textbook)**

Nasco's Worksheets for Algebra II and Precalculus (Joke workbook)

**Internet Website Reference Material: www.wolfram.com and
<http://majortests.com/sat/>**