

Pre-Calculus

Grades 11-12

2008

Course Description

Pre-Calculus reviews and introduces the same concepts necessary for success in the Calculus. The emphasis is on using and understanding mathematics. This is not a course where the student can expect to master mathematical skill for a short time only and expect to succeed. Students must develop the ability to analyze polynomial, rational, exponential and logarithmic functions (graphs, domain, range, maxima, minima, continuity and end behavior). They will become competent when dealing with matrices. We will develop other concepts necessary for the calculus to include methods for solving equations and inequalities, manipulating polynomials, irrational numbers, complex numbers, inverse functions and composite functions. Students must not only be able to solve equations and inequalities but understand what they are doing and why. They must grasp that mathematics exists as a pure abstract and as a tool to model real situations. Students will be tested on concepts introduced in the beginning of the semester throughout the semester. Their Calculus will require mastery of this semester course when they take the next course in high school or college as will their college science instructors.

The student will be able to :

1. Evaluate and solve equations by various methods, including factoring, applying the Quadratic Formula, and by completing the square.
2. Evaluate, graph, and solve linear and combined inequalities, as well as inequalities involving absolute values.
3. Apply laws of exponents to simplify, graph, and solve radical and exponential expressions and equations.
4. Understand functions and relations , domain, range and graphs of functions.
5. Graph equations of circles, identifying key points and properties of their graphs.
6. Apply properties of slope to graph, interpret graphs and write equations.
7. Graph the library of Graphs and apply transformations.
8. Perform mathematical operations on functions and form composite functions.
9. Graph and interpret graphs of quadratic and polynomial functions.
10. Determine and graph the inverse functions.
11. Determine real and complex zeros of polynomial functions.
12. Apply properties of logarithms and evaluate logarithms with various bases.
13. Evaluate, graph and convert between exponential and logarithmic functions.

14. Apply exponential and logarithmic functions to real life problem solving.

CUMULATIVE PROGRESS INDICATORS:

STANDARD 4.1 - MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.
3. Perform operations on matrices.
 - Addition and subtraction
 - Scalar multiplication
4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

STANDARD 4.2 - MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Geometric Properties

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a hallway, doorway, or have a ladder reach a certain height on a building).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.
 - Parallel lines . transversal, alternate interior angles, corresponding angles
 - Triangles
 - a. Conditions for congruence
 - b. Segment joining midpoints of two sides is parallel to and half the length of the third side

c. Triangle Inequality

- Minimal conditions for a shape to be a special quadrilateral
 - Circles . arcs, central and inscribed angles, chords, tangents
 - Self-similarity
4. Use reasoning and some form of proof to verify or refute conjectures and theorems.
- Verification or refutation of proposed proofs
 - Simple proofs involving congruent triangles
 - Counterexamples to incorrect conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.
3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.
- Fractals (e.g., Sierpinski's Triangle)
 - Patterns in areas and perimeters of self-similar figures
 - Outcome of extending iterative process indefinitely

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.
- Distance between two points
 - Midpoint and slope of a line segment
 - Finding the intersection of two lines
 - Lines with the same slope are parallel
 - Lines that are perpendicular have slopes whose product is -1
2. Show position and represent motion in the coordinate plane using vectors.
- Addition and subtraction of vectors

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
- Degree of accuracy of a given measurement tool
 - Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.
- Similar triangles

- Pythagorean theorem
 - Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.
- Approximation of area using grids of different sizes
 - Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
 - Estimation of area, perimeter, volume, and surface area

STANDARD 4.3 - MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.
 - Explicit formulas for n^{th} terms
 - Sums of finite arithmetic series
 - Sums of finite and infinite geometric series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.
 - Slope of a line or curve
 - Domain and range
 - Intercepts
 - Continuity
 - Maximum/minimum
 - Estimating roots of equations
 - Intersecting points as solutions of systems of equations
 - Rates of change
3. Understand and perform transformations on commonly-used functions.
 - Translations, reflections, dilations
 - Effects on linear and quadratic graphs of parameter changes in equations
 - Using graphing calculators or computers for more complex functions
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.
 - Linear vs. non-linear
 - Symmetry
 - Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.

- Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of

mailing a first-class letter over the past 200 years)

- Direct and inverse variation

- Absolute value

- Expressions, equations and inequalities

- Same function can model variety of phenomena

- Growth/decay and change in the natural world

- Applications in mathematics, biology, and economics (including compound interest)

2. Analyze and describe how a change in an independent variable leads to change in a dependent

one.

3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Evaluate and simplify expressions.

- Add and subtract polynomials

- Multiply a polynomial by a monomial or binomial

- Divide a polynomial by a monomial

2. Select and use appropriate methods to solve equations and inequalities.

- Linear equations . algebraically

- Quadratic equations . factoring (when the coefficient of x^2

is 1) and using the quadratic

formula

- All types of equations using graphing, computer, and graphing calculator techniques

3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including

those carried out by technology.

Standard 8.1 - Technology

All students will use computer applications to gather and organize information and to solve problems. Technology, any modification of the natural world designed by human beings to solve human problems, enhance human life, or extend human capability, was identified by the United States Department of Labor as an essential workplace competency in a 1992 report called the Secretary's Commission on Achieving Necessary Skills (SCANS). SCANS stated that students should be able to select equipment and tools, apply technology to specific tasks, and maintain and troubleshoot equipment. The Department of Education recognized its importance by including technology in the original cross-content workplace readiness standards. In keeping with today's technological society, technological literacy has been further emphasized by its inclusion

as a separate standards area which focuses on both computer and information literacy and technology education. Technology is evolving at an amazing rate, with both frequent advancements of existing technology and the creation of new technologies. All students must understand and be comfortable with the concepts and application of technology, not only in order to function in today's complex society, but also to become informed and productive adults of tomorrow. **Computer and Information Literacy** Computer and information literacy, which supports skills in information-gathering, information-organizing, and problem solving, has become critical for every student whether college- or workplace-bound. Colleges and employers are now demanding that students and employees possess a broad range of computer and information literacy proficiencies. More and more retail purchasing is being done on-line every year, and all but the most menial of positions now require a significant understanding of computer and information literacy. To ensure that students are computer literate, a separate standard that defines rigorous, in-depth learning has been included. The computer and information literacy standard is designed to be integrated and applied in all of the content areas of the Core Curriculum Content Standards. **Technology Education** The technology education standard was developed to ensure the literacy needed by all students to succeed in a highly technological world. Business and industry has clearly stated the need for technological skills in the workplace of the 21st Century. This standard is based on the Standards for Technological Literacy (STL): Content for the Study of Technology (ITEA, 2000), developed as part of the National Science Foundation (NSF)/National Aeronautics and Space Administration (NASA) funded by the Technology for All Americans (TfAA) project. A study by DeKlerk has found that students form negative attitudes about the technological world if there are no formal technological experiences during the early school years. This finding Technological Literacy 1 Technological Literacy is a great concern to New Jersey business and industry. Other cognitive research suggests that "design-based learning" is important. Early studies with design and technology curriculum indicate that students who learn important technological concepts develop positive attitudes about technology, math, science and learning in general. For these reasons, an introduction to technology education, including engineering and technological design, is an essential component of a thorough and efficient K-12 education. **Standards and Strands** There are two technological literacy standards, each of which has a number of lettered strands. The standards and strands include: **8.1 Computer and Information Literacy A. Basic Computer Tools and Skills** • Keyboarding • Word processing • Internet usage • Spreadsheets • Database concepts and usage • Publications and presentations

Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).

5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences.

UNIT 1 Graphs

- 1.1 Rectangular Coordinates
- 1.2 Graphs of equations
- 1.3 Lines and their equations
- 1.4 Linear Curve Fitting

Students will be able to

- 1 plot points and lines
- 2 determine parallel and perpendicular lines
- 3 determine equations of lines parallel or perpendicular to a given line through a given point
- 4 connect graphs to their equations
- 5 given a set of data determine a line of best fit

Instructional strategies

- Lecture
- Black/White &/or smart board work
- Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.
- Vocabulary

Alternative Assessment:

- Cooperative Learning
- Do-Now Problems
- Problem Solving
- Develop a drawing of word problems.
- Develop a chart of word problems and the equations that would solve them.
- Use of technology based resources
 - TI-83/84 and TI84 emulation software
 - smart board software
 - Geometer's Sketchpad

EVALUATION/ASSESSMENT OF STUDENTS:

SOLVING INEQUALITIES

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work

- Class Participation
- Alternative Assessment
 - Collect a set of data on height vs. arm length
 - Students develop word problems that can be represented by a straight line graph.
 - Students identify where the measuring of angles would benefit a project.

Unit Two – Functions and their graphs

- 2.1 functions
- 2.2 properties of functions
- 2.3 library of functions; Piecewise-Defined Functions
- 2.4 Graphing Techniques: Transformations
- 2.5 Operations on functions; Composite functions
- 2.6 Mathematical models: constructing functions

Students will be able to:

- 1 graph functions given their equation
- 2 determine intervals of increase and decrease
- 3 define functions over certain subintervals
- 4 $f \circ g, g \circ f, f \circ f, g \circ g$
- 5 construct equations to represent the activities identified in word problems

Instructional strategies

- Lecture
- Black/White &/or smart board work
- Use of Open-ended problems, written and oral exercises, and quantitative comparison activities
- Vocabulary

Alternative Assessment:

- Cooperative Learning
- Do-Now Problems
- Problem Solving
- Develop a drawing of word programs.
- Develop a chart of word problems and the equations that solve them.
- Use of technology based resources
 - TI-83/84 and TI84 emulation software
 - smart board software
 - Geometer's Sketchpad

EVALUATION/ASSESSMENT OF STUDENTS: SOLVING INEQUALITIES

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work
- Class Participation
- Alternative Assessment
 - define two sets of relatives and perform the four composite functions on them
 - students make a chart of equations and their graphs

Unit Three – Polynomial and Rational Functions

- 3.1 Quadratic Functions and Models
- 3.2 Polynomial Functions
- 3.3 Rational Functions and analyzing graphs
- 3.4 Polynomial and rational inequalities
- 3.5 The Real Zeros of a Polynomial Function
- 3.6 Complex Zeros and the Fundamental Theorem of Algebra

Students will be able to :

- 1 graph a quadratic function using its vertex, axis of symmetry and intercepts
- 2 graph a quadratic function using transformations
- 3 use the extrema values of a quadratic function to solve applied functions
- 4 use a graphing utility to confirm these data
- 5 identify polynomials by degree
- 6 identify the zeros of a polynomial and the multiplicity of the zeros
- 7 find the domain of a rational function
- 8 determine the vertical, horizontal and oblique asymptotes of a rational function
- 9 analyze the graph of rational functions
- 10 solve applied problems involving rational functions
- 11 solve polynomial and rational inequalities
- 12 use the remainder and factor theorems
- 13 determine the number of positive and negative real zeros
- 14 determine the real zeros of a function
- 15 use the intermediate value theorem
- 16 determine the complex zeros of a polynomial

- Lecture
- Black/White &/or smart board work
- Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.
- Vocabulary

Alternative Assessment:

- Cooperative Learning
- Do-Now Problems
- Problem Solving
- Develop a drawing of word programs.
- Develop a chart of word problems.
- Use of technology based resources
 - TI-83/84 and TI84 emulation software
 - smart board software
 - Geometer's Sketchpad

EVALUATION/ASSESSMENT OF STUDENTS:

SOLVING INEQUALITIES

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work
- Class Participation
- Alternative Assessment
 - Students compare work on homework and classwork
 - Students make a poster of families of polynomials and of their degree
 - Students identify where the
 - Research the life and work of Pascal

Unit Four – Exponential and Logarithmic Functions

- 4.1 one to one and inverse functions
- 4.2 exponential functions
- 4.3 logarithmic functions
- 4.4 properties of logarithms
- 4.5 logarithmic and exponential equations
- 4.6 compound interest

4.7 growth and decay; Newton's Law; logistic models

Students will be able to:

- 1 determine the inverse of a function
- 2 obtain the graph of the inverse function when the graph of the function is known
- 3 determine the inverse function f^{-1}
- 4 show functions to be one – to – one
- 5 evaluate and graph exponential functions
- 6 know the decimal equivalent of e correct to five decimal places
- 7 change exponential expressions to logarithmic functions and vv.
- 8 determine the domain and evaluate logarithmic functions
- 9 graph and solve logarithmic functions
- 10 write a logarithmic function as the sum or difference of logarithms
- 11 evaluate logarithms whose base is neither 10 nor e
- 12 solve logarithmic equations using properties of logarithms
- 13 solve exponential equations and the two equations together with a graphing utility
- 14 calculate effective rates of return
- 15 determine the present value of a lump sum of money
- 16 determine the time necessary for a lump sum to double or triple
- 17 find equations of populations that obey the law of uninhibited growth
- 18 find equations of populations that obey the law of decay
- 19 use logistic growth models

Lecture

Black/White &/or smart board work

Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.

Vocabulary

Alternative Assessment:

Cooperative Learning

Do-Now Problems

Problem Solving

Develop a drawing of word programs.

Develop a chart of word problems.

Use of technology based resources

○ TI-83/84 and TI84 emulation software

○ smart board software

○ Geometer's Sketchpad

EVALUATION/ASSESSMENT OF STUDENTS:

SOLVING INEQUALITIES

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work
- Class Participation
- Alternative Assessment
 - Committee work
 - Students use a team approach to sketch a logarithmic or exponential function
 - Research the development of the logistic growth model

Unit Five – Matrices

- 5.1 systems of linear equations with two and three variables
- 5.2 systems of linear equations: matrices and determinants
- 5.3 matrix algebra

Students will be able to:

- 1 solve systems of equation by substitution and elimination
- 2 identify inconsistent equations
- 3 express the solution of a system of dependent equations
- 4 write the augmented matrix of a system of linear equations
- 5 write a system from an augmented matrix
- 6 solve a system of linear equations using matrices

- Lecture
- Black/White &/or smart board work
- Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.
- Vocabulary

Alternative Assessment:

- Cooperative Learning
- Do-Now Problems
- Problem Solving
- Develop a drawing of word programs.
- Develop a chart of word problems.
- Use of technology based resources

- TI-83/84 and TI84 emulation software
- smart board software
- Geometer's Sketchpad

EVALUATION/ASSESSMENT OF STUDENTS:

SOLVING INEQUALITIES

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work
- Class Participation
- Alternative Assessment
 - Committee work
 - Students develop poster showing the steps of matrix augmentation
 - Students identify other uses of matrices
 - Research ibn al Khorizmi

EVALUATION/ASSESSMENT OF CURRICULUM:

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five Year Curriculum Review schedule.

RESOURCES/BIBLIOGRAPHY:

“New Jersey Core Curriculum Content Standards for Technological Literacy” New Jersey State Department of Education, 2004
 “New Jersey Mathematics Curriculum Framework”, Joseph G. Rosentein, Janet H. Caldwell, Warren D. Crown, 2004
 PRECALCULUS, the Sixth Edition, Michael Sullivan, Prentiss Hall, Englewood Cliffs, NJ