# Statistics and Probability
2008

**COURSE DESCRIPTION:**

The branch of mathematics known as Probability Theory originated in seventeenth century France when the two great French mathematicians, Blaise Pascal and Pierre de Fermat, corresponded over two problems from games of chance. Pascal and Fermat's approach to the problems influenced other early researchers such as Huygens, Bernoulli, and DeMoivre as they established a mathematical theory of probability. Today, probability theory is a broadly utilized branch of mathematics that is applied to every area of modern day activity. With applications in music, physics and in every daily experience from weather forecasting to predicting and evaluating the risks of new medical treatments; probability, combined with statistics is a critical tool for achieving success in numerous endeavors.

In many aspects of mathematics a formula or process is used to predict the future. Sometimes the prediction is exact, but often events take place randomly and it is not possible to predict exactly what will happen. A model must be constructed that takes this random nature of outcomes into account. It is the creation of such models that surround the fields of statistics and probability.

In particular, this course will first address simulating events, predicting their probability of occurrence. Within probability the course deals with randomness, probability models, and discrete and continuous random variables.

After covering many aspects of probability, the course will flow into using various techniques of the field of statistics to analyze data. The statistical portion of the course will cover organizing data, normal distributions, scatter plots, correlation, the least squares regression, and producing data from samples and experiments. The course will also address the potential pitfalls when interpreting statistics and relying on the statistical interpretations of others.

**CORE CURRICULUM CONTENT STANDARDS:**

**The Vision**

The vision of the mathematics standards is focused on achieving one crucial goal:
To enable ALL of New Jersey's children to acquire the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives. We want ALL students to achieve the standards. There may be exceptions, but those exceptions should be exceptional.

Perhaps the most compelling reason for this vision is that all of our children, as well as our state and our nation, will be better served by higher expectations, by curricula that go far beyond basic skills and include a variety of mathematical models, and by programs which devote a greater percentage of instructional time to problem-solving and active learning.

Many students respond to the traditional curriculum with boredom and discouragement. They feel that mathematics will never be useful in their lives, and they develop the perception that success in mathematics depends on some innate ability that they simply do not have. [1]We must overcome the feelings among students that they don't like mathematics, they don't need mathematics, and they can't do mathematics. Curricula that evoke these responses in students, curricula that assume student failure, are bound to fail; we need to develop curricula that assume student success.

Our curricula are often preoccupied with what national reports call "shopkeeper arithmetic" competency in the basic operations that were needed to run a small store several generations ago. The economy in which graduates of our schools will seek employment is more competitive than ever and is rapidly changing in response to advances in technology. To compete in today's global, information-based economy, students must be able to solve real problems, reason effectively, and make logical connections.

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[1] "Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents, and teachers all expect that most students can master mathematics if only they work hard enough. The record of accomplishment in these countries - and in some intervention programs in the United
States - shows that most students can learn much more mathematics than is commonly assumed in this country."
Everybody Counts, Mathematical Sciences Education Board, National Academy of Sciences (1989)

**STANDARD 4.4** (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS)
ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

Descriptive Statement: Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

Data Analysis (or Statistics): In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

Probability: Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rates are provided in terms of percentages. They should regularly engage in predicting and determining probabilities, often based on experiments(like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

Discrete Mathematics•Systematic Listing and Counting. Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

**STANDARD 8.1:** All students will use computer applications to gather and organize information to solve problems.

**STANDARD 9.1:** All Students will develop career awareness and planning, employability skills, and foundational knowledge necessary for success in the workplace.

**STANDARD 9.2:** All students will demonstrate critical life skills in order to be functional members of society.

# Cumulative Progress Indicators:

**(4.4A)** DATA ANALYSIS *Building upon knowledge and skills gained in preceding grades, by the end of* ***Grade 12****, students will:*

1. Use surveys and sampling techniques to generate data and draw conclusions about large groups. *Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling)

2. Evaluate the use of data in real-world contexts. *Accuracy and reasonableness of conclusions drawn, *Correlation vs. causation*, Bias in conclusions drawn (e.g., influence of how data is displayed), Statistical claims based on sampling,

3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.

4. Analyze data using technology, and use statistical terminology to describe conclusions.
- Measures of dispersion: variance, standard deviation, outliers
- Correlation coefficient
- Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)

5. Distinguish between randomized experiments and observational studies.

**(4.4C)**Building upon knowledge and skills gained in the preceding grades, by the end of Grade 12, students will:

1. Estimate probabilities and predict outcomes from actual data.

2. Understand sampling and recognize its role in statistical claims.

3. Evaluate bias, accuracy, and reasonableness of data in real-world contexts.

4. Understand and apply measures of dispersion and correlation.

5. Design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.

6. Make predictions using curve fitting and numerical procedures to interpolate and extrapolate from known data.

7. Use relative frequency and probability, as appropriate, to represent and solve problems involving uncertainty.

8. Use simulations to estimate probabilities.

9. Create and interpret discrete and continuous probability distributions, and understand their application to real-world situations.

10. Describe the normal curve in general terms, and use its properties to answer questions about sets of data that are assumed to be normally distributed.

11. Understand and use the law of large numbers (that experimental results tend to approach theoretical probabilities after a large number of trials).

**(8.1)**

**A** Basic Computer Skills and Tools

1. Use appropriate technology vocabulary.
2. Use common features of an operating system
3. Demonstrate effective input of text and data, using touch keyboarding with proper technique.
4. Input and access data and text efficiently and accurately through proficient use of other input devices, such as the mouse.
5. Create documents with advanced text-formats, graphics using word processing.
6. Create a file containing customized information by merging documents.
7. Construct a simple spreadsheet, enter data, and interpret the information.
8. Design and produce a basic multimedia project.
9. Plan and create a simple database, define fields, input data, and produce a report using sort and query.
10. Use network resources for storing and retrieving data.
11. Choose appropriate electronic graphic organizers to create, construct, or design a document.
12. Create, organize and manipulate shortcuts.

**B. Application of Productivity Tools**

**Social Aspects**

1. Demonstrate an understanding of how changes in technology impact the workplace and society.
2. Exhibit legal and ethical behaviors when using information and technology, and discuss consequences of misuse.
3. Explain the purpose of an Acceptable Use Policy and the consequences of inappropriate use of technology.
4. Describe and practice safe Internet usage.
5. Describe and practice "etiquette" when using the Internet and electronic mail.

**Information Access and Research**

6. Choose appropriate tools and information resources to support research and solve real world problems, including but not limited to:
   - On-line resources and databases
   - Search engines and subject directories
7. Evaluate the accuracy, relevance, and appropriateness of print and non-print electronic information sources.

**Problem Solving and Decision Making**

8. Use computer applications to modify information independently and/or collaboratively to solve problems.
9. Identify basic hardware problems and demonstrate the ability to solve common problems.
10. Determine when technology tools are appropriate to solve a problem and make a decision.

**(9.1)** Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

**A. Career Awareness/Preparation**

1. [Analyze] Re-evaluate personal interests, abilities, and skills through various measures including self assessments.
2. Evaluate academic and career skills needed in various career clusters.
3. Analyze factors that can impact an individual's career.
4. Review and update one's career plan and include in a portfolio.
5. Research current advances in technology that apply to a selected occupational career cluster.

**B. Employability Skills**

1. Assess personal qualities that are needed to obtain and retain a job related to career clusters.
2. Communicate and comprehend written and verbal thoughts, ideas, directions, and information relative to educational and occupational settings.
3. Select and utilize appropriate technology in the design and implementation of teacher approved projects relevant to occupations and/or higher educational settings.
4. Evaluate the following academic and career skills as they relate to home, school, community, and employment:
   - Communication
   - Punctuality
   - Time management
   - Organization
   - Decision making
   - Goal setting
   - Resources allocation
   - Fair and equitable competition
   - Safety
   - Employment application skills
   - Teamwork
5. Demonstrate teamwork and leadership skills that include student participation in real world applications of career and technical education skills [through student organizations].

**All students electing further study in career and technical education will also:**

6. Participate in a structured learning experience that demonstrates interpersonal communication, teamwork, and leadership skills.
7. Participate in simulated industry assessments when and where appropriate.
8. Prepare industry-specific technical reports/projects that incorporate graphic aids when and where appropriate.
9. Demonstrate occupational health and safety skills related to industry-specific activities.

**(9.2)** Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

**A. Critical Thinking**
1. Apply communications and data analysis to the problem-solving and decision making processes [to] in a variety of life situations.
2. Describe and apply constructive responses to criticism.
3. Apply the use of symbols, pictures, graphs, objects, and other visual information to a selected project in academic and/or occupational settings.
4. Recognize bias, vested interest, stereotyping and the manipulation and misuse of information while formulating solutions to problems that interfere with attaining goals.
5. Apply knowledge and skills needed to use various means of transportation within a community.

**B. Self-Management**
1. Revise and update the personal growth plan to address multiple life roles.
2. Apply project planning and management skills in academic and/or occupational settings.
3. Compare and contrast methods for maximizing personal productivity.

**C. Interpersonal Communication**
1. Model interpersonal and effective conflict resolution skills [to peers].
2. Communicate effectively in a variety of settings with a diverse group of people.

**D. Character Development and Ethics**
1. Analyze how character influences work performance.
2. Identify and research privileges and duties of citizens in a democratic society.
3. Discuss consequences and sanctions when on-the-job rules and laws are not followed.
4. Compare and contrast a professional code of ethics or code of conduct from various work fields and discuss similarities and differences.
5. Apply a professional code of ethics to a workplace problem or issue.

SUGGESTED ACTIVITIES THAT ADDRESS THE DATA ANALYSIS AND PROBABILITY STANDARD:

**(4.4A)**

- Discuss the need for statistics.
- Discuss statistical problem solving methodologies.
- Explain how to organize raw data into an array.
- Discuss the types of tables and charts that are used to honestly analyze and present numeric facts and how measures of central tendency and dispersion help provide information about a set of data
- Demonstrate methods of presenting and identifying how people's biases, conscious or not, can affect how they represent data.

Relating to predictions using curve fitting and numerical procedures to interpolate and extrapolate from known data:

- Demonstrate how to organize raw data into an array and how to construct and interpret a frequency distribution.
- Use the TI-83 graphical calculators to find methods of best fit of algebraic, trigonometric and exponential curves to data points.
- Illustrate methods by hand and with the calculator as well as the Fathom software, for displaying data as histograms, frequency polygons, stem and leaf plots, dot and box plots.

Relating to using relative frequency and probability to represent and solve problems involving uncertainty:

- Differentiate between continuous and discrete random variables.
- Show a method for computing the expected value and standard deviation of a discrete random variable.
- Use the Fathom software for these representations and solutions.

Relating to creating and interpreting discrete and continuous probability distributions and understanding their application to real world situations as well as using such measures of central tendency as the normal curve and discussing the law of large numbers:

- Demonstrate a probability distribution.
- Discuss evaluating the data in a probability distribution.
- Illustrate the computing of such central tendency measures as the arithmetic mean, median and mode.
- Show how to calculate the dispersion measures of range, mean absolute deviation and standard deviation.
- Discuss the characteristics of the normal curve and its percentages of data within one, two or three standard deviations.
- Discuss the effect of large numbers in data sets.
- Show the effects of allowing a large number of trials to approach the normal curve. Include a discussion of infinity.

**(4.4C)**

- Demonstrate and discuss sample spaces.
- Define and illustrate frequency, relative frequency and probability.
- Define and obtain for a sample space the possible outcomes.
- Define and discuss an event as a subset of a sample space.
- Use the software package Fathom to generate data and visually present probabilities on the smart board or as power point.
- Demonstrate and define mutually exclusive events.
- Collect data from classmates and/or school staff for a particular situation and present as a project the probabilities involved.
- Define and discuss equally likely outcomes.
- Express probabilities in a probability distribution.
- Demonstrate the use of the addition principle for mutually exclusive events.
- Demonstrate and verify the rule for complements.
- In a Venn Diagram discuss the addition principle.
- Apply the addition principle to social science studies.
- Define and discuss empirical probabilities.
- Demonstrate counting techniques.
- Show several controlled experiments.
- Use the product rule for independent events.
- Demonstrate Baye's Formula

**(8.1)**

Use a computer to research, prepare, and present individual or group projects relating to the above tasks.

## INSTRUCTIONAL STATEGIES:

Included but not limited to, would be the following:

- Use of committee settings, randomly selected from the class for purposes of drawing inferences, rolling of dice, flipping of coins, etc., and the subsequent presentations of the data.
- A small amount of lecture at the outset or conclusion of such group work
- Groups gathering data from schoolmates and/or school staff
- Using the software Fathom, to give data for many situations, allow students to select their social/scientific concern and to present a complete course-length project on their data
- Making use of the overhead demonstration of the graphical calculator as well as allowing groups of students to use the graphical calculator as a demonstration device in addition to number crunching
- Groups will be using this capability also

## EVALUATION/ASSESSMENT OF STUDENTS:

Included but not limited to would be the following:

- Announced tests
- Unannounced quizzes
- Projects
- Evaluations of group and individual presentations

## EVALUATION/ASSESSMENT OF CURRICULUM

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule.

## RESOURCES/BIBLIOGRAPHY:


- Statistics, Sanders and Schmidt, McGrawHill
- Functions, Statistics and Trigonometry, University of Chicago School Mathematics Project
- Finite Mathematics with Applications, Zitarelli and Coughlin, HBJ
- Introduction to Statistics and Data Analysis, Peck, Olsen, Devore, Duxsbury
- The Practice of Statistics, Yates, Moore, McCabe, Freeman
- Advanced Mathematics, Brown, Houghton Mifflin
- Probability, Kinney, Wiley
- Understanding Basic Statistics, Brase & Brase, Houghton Mifflin
- Rutgers' Leadership Program in Discrete Mathematics
- The Mathematics Forum
- Eisenhower Consortium for Better Schools