

Trigonometry

Grades 11-12

2008

Course Description:

Trigonometry review and introduces some concepts necessary for success in the Calculus. The emphasis is on using and understanding mathematics. This is not a course where the student can expect to master a mathematical skill for a short time only and expect to succeed. Students must develop the ability to analyze trigonometric functions (graphs, domain, range, maxima, minima, continuity and end behavior). They will be able to understand and use a body of trigonometric identities and use them to verify others. They will become competent when dealing with trigonometric, rational and polar functions. We will develop other concepts necessary for the calculus to include methods of solving equations and inequalities and when manipulating functions. These concepts from the Pre-Calculus semester course will be reviewed during the Trigonometry semester course. Students need to grasp that mathematics exists as a pure abstract and as a tool to model real situations. They will be tested throughout the semester on concepts taught throughout the course. Their Calculus course, whether taught at this institution or in a school of higher education setting will require mastery of trigonometry as will their courses in college sciences.

CUMULATIVE PROGRESS INDICATORS:

STANDARD 4.1 - MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.
3. Perform operations on matrices.
 - Addition and subtraction
 - Scalar multiplication
4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation,
and determine whether the error is within acceptable tolerance limits.

STANDARD 4.2 - MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Geometric Properties

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a hallway, doorway, or have a ladder reach a certain height on a building).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.
 - Parallel lines . transversal, alternate interior angles, corresponding angles
 - Triangles
 - a. Conditions for congruence
 - b. Segment joining midpoints of two sides is parallel to and half the length of the third side
 - c. Triangle Inequality
 - Minimal conditions for a shape to be a special quadrilateral
 - Circles . arcs, central and inscribed angles, chords, tangents
 - Self-similarity
4. Use reasoning and some form of proof to verify or refute conjectures and theorems.
 - Verification or refutation of proposed proofs
 - Simple proofs involving congruent triangles
 - Counterexamples to incorrect conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.
3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.
 - Fractals (e.g., Sierpinski's Triangle)
 - Patterns in areas and perimeters of self-similar figures
 - Outcome of extending iterative process indefinitely

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.
 - Distance between two points
 - Midpoint and slope of a line segment
 - Finding the intersection of two lines
 - Lines with the same slope are parallel
 - Lines that are perpendicular have slopes whose product is -1
2. Show position and represent motion in the coordinate plane using vectors.
 - Addition and subtraction of vectors

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
 - Degree of accuracy of a given measurement tool
 - Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.
 - Similar triangles
 - Pythagorean theorem
 - Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.
 - Approximation of area using grids of different sizes
 - Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
 - Estimation of area, perimeter, volume, and surface area

STANDARD 4.3 - MATHEMATICS

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.
 - Explicit formulas for n^{th} terms
 - Sums of finite arithmetic series
 - Sums of finite and infinite geometric series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using

appropriate graphing technologies.

- Slope of a line or curve
 - Domain and range
 - Intercepts
 - Continuity
 - Maximum/minimum
 - Estimating roots of equations
 - Intersecting points as solutions of systems of equations
 - Rates of change
3. Understand and perform transformations on commonly-used functions.
- Translations, reflections, dilations
 - Effects on linear and quadratic graphs of parameter changes in equations
 - Using graphing calculators or computers for more complex functions
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.
- Linear vs. non-linear
 - Symmetry
 - Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.

- Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of

mailing a first-class letter over the past 200 years)

- Direct and inverse variation
- Absolute value
- Expressions, equations and inequalities
- Same function can model variety of phenomena
- Growth/decay and change in the natural world
- Applications in mathematics, biology, and economics (including compound interest)

2. Analyze and describe how a change in an independent variable leads to change in a dependent

one.

3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Evaluate and simplify expressions.

- Add and subtract polynomials
- Multiply a polynomial by a monomial or binomial
- Divide a polynomial by a monomial

2. Select and use appropriate methods to solve equations and inequalities.

- Linear equations . algebraically
- Quadratic equations . factoring (when the coefficient of x

is 1) and using the quadratic formula

- All types of equations using graphing, computer, and graphing calculator techniques
3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

STANDARD 4.5 – MATHEMATICS

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - Open-ended problems
 - Non-routine problems
 - Problems with multiple solutions
 - Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., .try a simpler problem. or .make a diagram.) to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
 - Reading and writing
 - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world

languages and social studies standards).

6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
 - Counterexamples as a means of disproving conjectures
 - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
 - Concrete representations (e.g., base-ten blocks or algebra tiles)
 - Pictorial representations (e.g., diagrams, charts, or tables)
 - Symbolic representations (e.g., a formula)
 - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

Standard 8.1 - Technology

All students will use computer applications to gather and organize information and to solve problems. Technology, any modification of the natural world designed by human beings to solve human problems, enhance human life, or extend human capability, was identified by the United States Department of Labor as an essential workplace competency in a 1992 report called the Secretary's Commission on Achieving Necessary Skills (SCANS). SCANS stated that students should be able to select equipment and tools, apply technology to specific tasks, and maintain and troubleshoot equipment. The Department of Education recognized its importance by including technology in the original cross-content workplace readiness standards. In keeping with today's technological society, technological literacy has been further emphasized by its inclusion as a separate standards area which focuses on both computer and information literacy and technology education. Technology is evolving at an amazing rate, with both frequent advancements of existing technology and the creation of new technologies. All students must understand and be comfortable with the concepts and application of technology, not only in order to function in today's complex society, but also to become informed and productive adults of tomorrow. **Computer and Information Literacy** Computer and information literacy, which supports skills in information-gathering, information-

organizing, and problem solving, has become critical for every student whether college- or workplace-bound. Colleges and employers are now demanding that students and employees possess a broad range of computer and information literacy proficiencies. More and more retail purchasing is being done on-line every year, and all but the most menial of positions now require a significant understanding of computer and information literacy. To ensure that students are computer literate, a separate standard that defines rigorous, in-depth learning has been included. The computer and information literacy standard is designed to be integrated and applied in all of the content areas of the Core Curriculum Content Standards. **Technology Education** The technology education standard was developed to ensure the literacy needed by all students to succeed in a highly technological world. Business and industry has clearly stated the need for technological skills in the workplace of the 21st Century. This standard is based on the Standards for Technological Literacy (STL): Content for the Study of Technology (ITEA, 2000), developed as part of the National Science Foundation (NSF)/National Aeronautics and Space Administration (NASA) funded by the Technology for All Americans (TfAA) project. A study by DeKlerk has found that students form negative attitudes about the technological world if there are no formal technological experiences during the early school years. This finding Technological Literacy 1 Technological Literacy is a great concern to New Jersey business and industry. Other cognitive research suggests that "design-based learning" is important. Early studies with design and technology curriculum indicate that students who learn important technological concepts develop positive attitudes about technology, math, science and learning in general. For these reasons, an introduction to technology education, including engineering and technological design, is an essential component of a thorough and efficient K-12 education. **Standards and Strands** There are two technological literacy standards, each of which has a number of lettered strands. The standards and strands include: **8.1 Computer and Information Literacy A. Basic Computer Tools and Skills** • Keyboarding • Word processing • Internet usage • Spreadsheets • Database concepts and usage • Publications and presentations

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences.

Unit 1 Trigonometric functions

- 1.1 Angles and their measure
- 1.2 Trigonometric Functions
- 1.3 Trigonometric Function properties
- 1.4 Graphs of sine and cosine functions
- 1.5 Graphs of tangents, cotangents, cosecant, and secant functions
- 1.6 Phase shift for sine, cosine and tangent

Students will be able to :

- 1 Convert between degrees, minutes and seconds, and decimal forms for angles
- 2 Determine the arc length of a circle
- 3 Convert from degrees to radians and from radians to degrees
- 4 Determine the area of a sector of a circle
- 5 Determine the linear velocity of a object traveling in circular motion
- 6 Determine the exact values of the trigonometric functions using a point on the unit circle
- 7 Determine the exact values for the trigonometric functions of quadrantal angles
- 8 Determine the exact values of the trigonometric functions of $\pi/4$, $\pi/3$ and $\pi/6$
- 9 Find exact values to integral multiples of the measures in $(\pi/4, \pi/3, \pi/6)$
- 10 Determine the domain and range of the trigonometric functions
- 11 Determine the period of the trigonometric functions
- 12 Determine the signs of the trigonometric functions
- 13 Find the values of the trigonometric functions utilizing the fundamental identities
- 14 Use the even and odd properties to find the exact values of the trigonometric functions
- 15 Graph transformations of the sine function
- 16 Graph transformations of the cosine function
- 17 Determine the amplitude and period of sinusoidal functions
- 18 Graph functions like $y = A \sin (\omega x)$ where ω means omega
- 19 Graph transformations of the tangent and cotangent functions
- 20 Graph transformations of the cosecant and secant functions
- 21 Determine the phase shift of a sinusoidal function
- 22 Graph sinusoidal functions like $y = A \sin (\omega x - p)$ where p means phi

Lecture

Black/White &/or smart board work

Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.

Vocabulary

Alternative Assessment:

- Cooperative Learning
- Do-Now Problems
- Problem Solving
- Develop a drawing of word programs.
- Develop a chart of word problems.
- Use of technology based resources
 - TI-83/84 and TI84 emulation software
 - smart board software
 - Geometer's Sketchpad

**EVALUATION/ASSESSMENT OF STUDENTS:
SOLVING INEQUALITIES**

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work
- Class Participation
- Alternative Assessment
 - Committee work
 - independent study
 - Students identify where the Trigonometry began
 - Research the early people who developed trigonometry

Unit II Analytic Trigonometry

- 2.1 inverse sine, cosine and tangent
- 2.2 inverse trigonometric functions
- 2.3 trigonometric identities
- 2.4 sum and different formulas
- 2.5 double-angle and half-angle formulas
- 2.6 trigonometric equations

Students will be able to:

- 1 find the exact value of the inverse sine, cosine and tangent functions
- 2 find an approximate value of the inverse sine, cosine and tangent functions
- 3 establish identities
- 4 use sum and difference rules to find exact values
- 5 use sum and difference rules to establish identities
- 6 use sum and difference rules involving inverse trigonometric functions

- 7 use double angle formulas to determine exact values
- 8 use double angle and half angle formulas to establish identities
- 9 use half angle formulas to determine exact values

- Lecture
- Black/White &/or smart board work
- Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.
- Vocabulary

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SOLVING INEQUALITIES

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 - Word Problems
 - Quantitative Comparison Questions
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- Homework
- Seat Work
- Class Participation
- Alternative Assessment
 - Listed Above
 - Students develop a list of the trigonometric identities so far
 - Students identify the applications of trigonometry from the text or other published material.

Unit III Applications of Trigonometric Functions

- 3.1 right triangle trigonometry
- 3.2 the law of sines
- 3.3 the law of cosines

3.4 the area of a triangle

Students will be able to:

- 1 determine the value of trigonometric functions of acute angles
- 2 use the complementary angle theorem
- 3 solve right triangles
- 4 solve applied problems
- 5 solve side angle angle and angle side angle triangles
- 6 solve side side angle triangles
- 7 solve side angle side triangles
- 8 solve side side side triangles
- 9 determine the area of side angle side triangles
- 10 determine the area of side side side triangles

- Lecture
- Black/White &/or smart board work
- Use of Open-ended problems, written and oral exercises, and quantitative comparison activities.
- Vocabulary

Alternative Assessment:

- Cooperative Learning
- Do-Now Problems
- Problem Solving
- Develop a drawing of word programs.
- Develop a chart of word problems.
- Use of technology based resources
 - TI-83/84 and TI84 emulation software
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EVALUATION/ASSESSMENT OF STUDENTS:

SOLVING INEQUALITIES

- Teacher generated quizzes and tests.
 - Multiple Choice Questions
 - Open-ended Questions
 - Writing Exercises
 - Word Problems
 - Quantitative Comparison Questions
- Book generated activities, quizzes, and tests.
- Homework
- Seat Work
- Class Participation

□ Alternative Assessment

- Committee Work
- Students make a poster of the three types of angle configurations and calculations of parts and area
- Students write an application of the trigonometry in this chapter to the real world other than those in the text

This course of study will be assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review Schedule.

Resources/Bibliography:

“New Jersey Core Curriculum Content Standards for Technological Literacy” New Jersey State

Department of Education, 2004

“New Jersey Mathematics Curriculum Framework”, Joseph G. Rosentein, Janet H. Caldwell, Warren D. Crown, 2004

PRECALCULUS, Paul Sullivan, Edition Six, Prentiss Hall Publishing, Upper Saddle River, NJ 07458