

Math HSPA Review
7/23/03

COURSE DESCRIPTION:

Mathematical Problem Solving is a course for students who have not made satisfactory progress towards passing the New Jersey High School Proficiency Assessment(HSPA) in mathematics. Students enrolled in this course during their senior year use this time to work on their Special Review Assessment(SRA) while reviewing for the next administration of the HSPA. Students who successfully pass the mathematics portion of the HSPA are permitted to drop this course at the start of the second semester.

CORE CURRICULUM CONTENT STANDARDS:

(4.1) Number and Numerical Operations

All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

(4.2) Geometry and Measurement

All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.

(4.3) Patterns and Algebra

All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

(4.4) Data Analysis, Probability, and Discrete Mathematics

All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

(4.5) Mathematical Processes

All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

CUMULATIVE PROGRESS INDICATORS:

(4.1)

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12**, students will:

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and

- negative exponents.
3. Perform operations on matrices.
 - a. Addition and subtraction
 - b. Scalar multiplication
 4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

(4.2)

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12**, students will:

A. Geometric Properties

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.
 - Parallel lines - transversal, alternate interior angles, corresponding angles
 - Triangles
 - a. Conditions for congruence
 - b. Segment joining midpoints of two sides is parallel to and half the length of the third side
 - c. Triangle Inequality
 - Minimal conditions for a shape to be a special quadrilateral
 - Circles - arcs, central and inscribed angles, chords, tangents
 - Self-similarity
4. Use reasoning and some form of proof to verify or refute conjectures and theorems.
 - Verification or refutation of proposed proofs
 - Simple proofs involving congruent triangles
 - Counterexamples to incorrect conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using

software as an aid to visualization.

3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.

Fractals (e.g., Sierpinski's Triangle)

Patterns in areas and perimeters of self-similar figures

Outcome of extending iterative process indefinitely

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.

Distance between two points

Midpoint and slope of a line segment

Finding the intersection of two lines

Lines with the same slope are parallel

Lines that are perpendicular have slopes whose product is 1

2. Show position and represent motion in the coordinate plane using vectors.

Addition and subtraction of vectors

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.

Degree of accuracy of a given measurement tool

Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.

Similar triangles

Pythagorean theorem

Right triangle trigonometry (sine, cosine, tangent)

2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.

Approximation of area using grids of different sizes

Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets

Estimation of area, perimeter, volume, and surface area

(4.3)

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12**, students will:

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.

Explicit formulas for n th terms

Sums of finite arithmetic series

Sums of finite and infinite geometric series

2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.

- Slope of a line or curve

- Domain and range

- Intercepts

- Continuity

- Maximum/minimum

- Estimating roots of equations

- Intersecting points as solutions of systems of equations

- Rates of change

3. Understand and perform transformations on commonly-used functions.

- Translations, reflections, dilations

- Effects on linear and quadratic graphs of parameter changes in equations

- Using graphing calculators or computers for more complex functions

4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.

- Linear vs. non-linear

- Symmetry

- Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.

- Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)

- Direct and inverse variation

- Absolute value

- Expressions, equations and inequalities

- Same function can model variety of phenomena

- Growth/decay and change in the natural world

- Applications in mathematics, biology, and economics (including compound

interest)

2. Analyze and describe how a change in an independent variable leads to change in a dependent one.
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Evaluate and simplify expressions.
 - Add and subtract polynomials

- Multiply a polynomial by a monomial or binomial
- Divide a polynomial by a monomial
- 2. Select and use appropriate methods to solve equations and inequalities.
 - Linear equations - algebraically
 - Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula
 - All types of equations using graphing, computer, and graphing calculator techniques
- 3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

(4.4)

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 12**, students will:

A. Data Analysis

1. Use surveys and sampling techniques to generate data and draw conclusions about large groups.
 - Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling)
2. Evaluate the use of data in real-world contexts.
 - Accuracy and reasonableness of conclusions drawn
 - Bias in conclusions drawn (e.g., influence of how data is displayed)
 - Statistical claims based on sampling
3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.
4. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.
5. Analyze data using technology, and use statistical terminology to describe conclusions.
 - Measures of dispersion: variance, standard deviation, outliers
 - Correlation coefficient
 - Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)

B. Probability

1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
2. Use concepts and formulas of area to calculate geometric probabilities.
3. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.
4. Determine probabilities in complex situations.
 - Conditional events

Complementary events

Dependent and independent events

5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.
6. Understand and use the "law of large numbers" (that experimental results tend to approach theoretical probabilities after a large number of trials).

C. Discrete Mathematics-Systematic Listing and Counting

1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students).
2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.
3. Justify solutions to counting problems.
4. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability.

D. Discrete Mathematics-Vertex-Edge Graphs and Algorithms

1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems.
 - Circuits that include every edge in a graph
 - Circuits that include every vertex in a graph
 - Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring
 - Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures)
2. Explore strategies for making fair decisions.
 - Combining individual preferences into a group decision (e.g., determining winner of an election or selection process)
 - Determining how many Student Council representatives each class (9 th , 10 th , 11 th , and 12 th grade) gets when the classes have unequal sizes (apportionment)

(4.5)

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - Open-ended problems
 - Non-routine problems
 - Problems with multiple solutions
 - Problems that can be solved in several ways

3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
 - Reading and writing
 - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
 - Counterexamples as a means of disproving conjectures
 - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
 - Concrete representations (e.g., base-ten blocks or algebra tiles)
 - Pictorial representations (e.g., diagrams, charts, or tables)
 - Symbolic representations (e.g., a formula)
 - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.

2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences.

SUGGESTED ACTIVITIES THAT ADDRESS THESE STANDARDS MAY INCLUDE BUT ARE NOT LIMITED TO:

(4.1)

Extending understanding of real number system to include rational and irrational numbers.

Evaluating expressions containing powers, roots, and factorials.

Applying absolute values, exponents, and approximations in real-life situations.

Translating numbers between standard notation and scientific notation.

Applying reflexive, transitive, and symmetric properties.

Simplifying expressions using the associative and commutative properties of arithmetic operations.

Applying primes, factors, and multiples in real - life situations.

Identifying equivalent and nonequivalent forms of fractions, decimals, and percents.

Demonstrating an understanding of the relationships between ratios, proportions, and percents.

(4.2)

Describing and giving examples of geometric terms.

Applying properties, definitions, and relationships to identify and classify two-dimensional shapes.

Applying properties, definitions, and relationships to identify and classify three-dimensional shapes.

Identifying the relationships between geometric figures.

Using the rectangular coordinate system to determine the effects on figures of transformations.

Developing and applying strategies for determining area and surface area.

Using vectors to show the position of an object.

Developing and applying strategies for determining volume.

Solving problems by applying the Pythagorean Theorem.

Applying basic trig ratios to solve problems.

(4.3)

Constructing, recognizing, and extending patterns.

Evaluating algebraic expressions using independent and dependent variables.

Applying algebraic operations to solve linear equations that reflect real-life situations.

Applying algebraic operations to solve inequalities that reflect real-life situations.

Using domain and range of relations and functions to solve problems.

Finding and graphing the slope of a line.

Applying basic transformations to graphs of functions.

(4.4)

Determining the probability of a simple and compound event.
Determining measures of central tendencies, range, rank, and frequency.
Selecting appropriate graphical representations of statistical measure.
Determining the number of possible combinations and outcomes by using tree diagrams.
Representing information using networks.
Analyzing and applying iterative processes to solve problems.(e.g. fractals)
Analyzing and applying recursive processes to solve problems.(e.g. compound interest)

(4.5)

Organizing information using illustrations, charts or graphs, discovering patterns, and arranging data.
Calculating with and using manipulatives.
Estimating values.
Substituting simpler numbers.
Translating sentences into equations.
Applying formulas, definitions, and rules.
Working in reverse.
Constructing proportions or ratios.

INSTRUCTIONAL STRATEGIES:

Instructional strategies may include, but are not limited to:

- Class lessons and instruction
- Technological applications
- Analysis of problem solving techniques
- Evaluation and modification of individual testing skills
- Co-operative learning exercises
- Use of various manipulatives

EVALUATION/ASSESSMENT OF STUDENTS:

Students will be evaluated/assessed by the following methods:

- Written tests/quizzes
- Practice problems - multiple choice and free response
- Completion of computer produced sample problems/tests
- HSPA retake(s)
- Completion of SRA if needed

EVALUATION/ASSESSMENT OF CURRICULUM:

This course of study will be evaluated/assessed by instructional staff during the first year of implementation for the purpose of necessary revision at the end of the first year. In addition, this course of study will be reviewed according to the Five-Year Curriculum Review schedule (see attached).

RESOURCES/BIBLIOGRAPHY:

- Internet Testing Services - PLATO, Learning Express Online
- HSPA Success in Mathematics; Steck-Vaughn/Berrent,2000
- Measuring Up on the New Jersey HSPA; The Peoples Publishing Group,Inc.,2000

