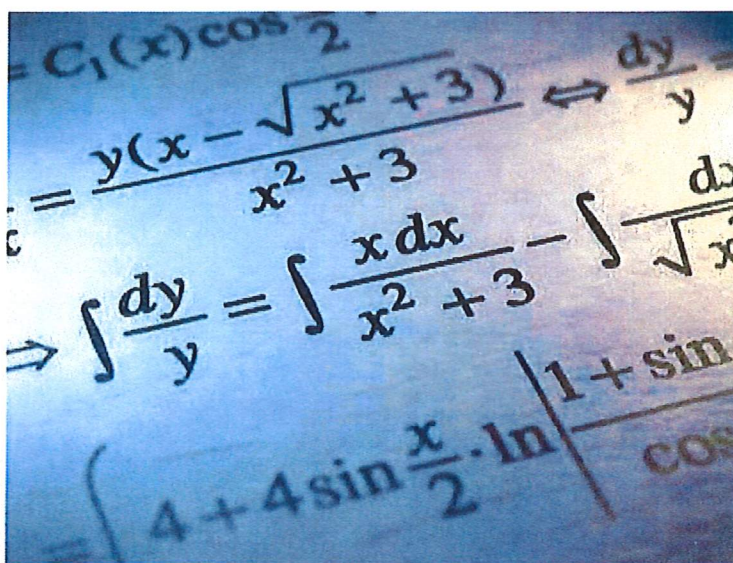


## SUMMER PACKET FOR AP CALCULUS



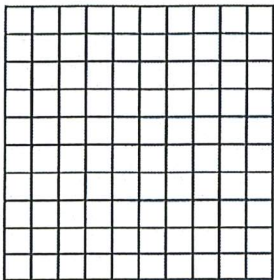
The image shows a close-up of handwritten mathematical work on a blue background. The equations are:

$$= C_1(x) \cos^2$$
$$= \frac{y(x - \sqrt{x^2 + 3})}{x^2 + 3} \Leftrightarrow \frac{dy}{y} =$$
$$\Rightarrow \int \frac{dy}{y} = \int \frac{x dx}{x^2 + 3} - \int \frac{dx}{\sqrt{x^2}}$$
$$= \left( 4 + 4 \sin \frac{x}{2} \cdot \ln \left| \frac{1 + \sin}{\cos} \right. \right)$$

Name: \_\_\_\_\_

1. Simplify  $\frac{5(x+h)^3 - 5x^3}{h}$ .

2. Sketch the graph of the piecewise function.  $f(x) = \begin{cases} \sqrt{x-2} + 1, & x > 2 \\ \frac{1}{2}x - 1, & 0 < x \leq 2 \\ x^2 - 1, & x \leq 0 \end{cases}$



3. Find the domain and range of each function. Find any symmetry (with respect to y-axis or origin).

a).  $f(x) = -3(x+2)^2 - 1$

b).  $g(x) = 2 \cos(x) + 3$

c).  $h(x) = \frac{x}{x^2 - 4}$

4. Factor:  $4x^4 - 19x^3 - 5x^2$

5. Solve the polynomial inequality:  $2x^3 - 11x^2 - 6x \leq 0$

6. Find y- and x-intercepts of the line which can be modeled by the equation  $\frac{A}{B}x + \frac{1}{C}y = 5$ . (A, B, C – are constants)

7. The equation of line L is  $8x - 14y = -3$ .

a) For what value of  $k$  is the graph of  $kx - 7y + 10 = 0$  parallel to line L?

b) For what value of  $k$  is the graph of  $kx - 7y + 10 = 0$  perpendicular to line L?

8. Write as a logarithm of a single expression:  $\frac{1}{2} \log_b(3x+1) - \frac{2}{3} \log_b(1-9x^2) + \log_b(1-x)$

9. Solve for  $x$ :  $\frac{1}{\sqrt{x-2}} - \frac{2(x-2)^{1/2}}{x+1} = 0$

10. Solve for  $t$ :  $\frac{d}{t+r} = \frac{5}{t}$

11. Solve for  $y'$ :  $3xy' - 2yy' = k$

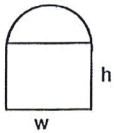
12. For  $f(x) = \frac{1}{x^2}$ , find  $\frac{f(x+h) - f(x)}{h}$ .

13. Evaluate the expression at the specified value of  $x$ .

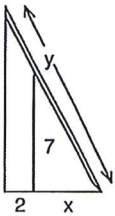
$$(3x^3 + 3)^{-\frac{2}{3}}, \quad x = 2$$

14. At what points do the curves  $f(x) = 2x^2 + x - 5$  and  $g(x) = x^2 - x + 3$  intersect?

15. Find an expression for the area of the window as shown. It consists of a rectangle of height  $h$  and width  $w$ , with a semicircle mounted on top of the rectangle.



16. A ladder is leaning against a wall and touches the top of a 7' fence which is 2' away from the wall. Express  $y$ , the length of the ladder required, as a function of  $x$ , the distance along the ground from the ladder to the fence.

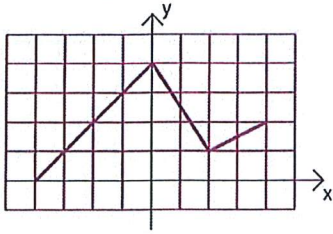


17. For  $f(x) = \sqrt{x^2 + 1}$  and  $g(x) = \frac{1}{x}$ , find the functions  $(f+g)(x)$ ,  $(f \cdot g)(x)$ ,  $(f \circ g)(x)$ ,  $(g \circ f)(x)$ , and  $(f \circ f)(x)$ .

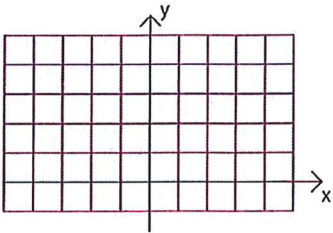
18. Given  $F(x) = \cos^2(x+9)$ , find the functions  $f$ ,  $g$ , and  $h$ , such that  $F(x) = f(g(h(x)))$

19. A ship is anchored five miles from the nearest point  $P$  on shore. A person plans to row a straight line from the ship to a point  $x$  miles downstream from  $P$ . Find an expression for the distance the person will have to row.

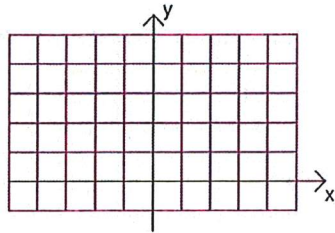
20. The graph of  $f$  is given. Draw the graphs of the following functions.



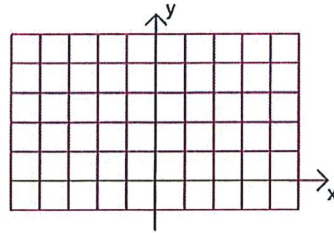
a)  $y = f(x+1)$



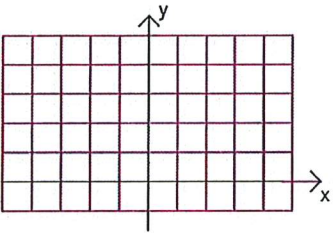
b)  $y = f(x)+1$



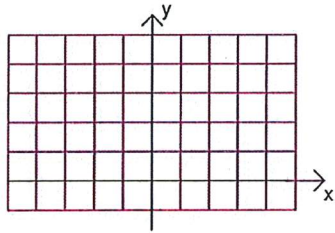
c)  $y = \frac{1}{2}f(x)+2$



d)  $y = -f(x)+3$



b)  $y = f(2x)$



21. Write an equation for the line  $y = f(x)$ , where  $f$  has the following values:

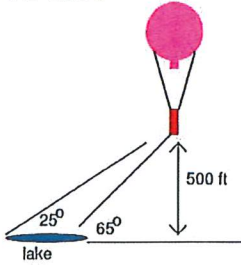
$x$	-2	2	4
$f(x)$	4	2	1

22. Find the points of intersections of the two curves  $f(x) = 2^x$  and  $g(x) = 3$ .

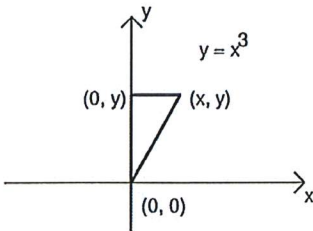
23. Find exact value of the expression  $5^{\log_5 6 + \log_5 7}$

24. Use algebra to solve:  $\log_3(x+1) + \log_3(x-1) = 2$

25. From a stationary hot air balloon 500 ft above the ground, two sightings of a lake are made. How long is the lake?



26. A right triangle has one vertex on the graph of  $y = x^3$ ,  $x > 0$ , at  $(x, y)$ , another at the origin, and the third on the positive y-axis at  $(0, y)$ , as shown. Express the area  $A$  of the triangle as a function of  $x$ .



27. Determine whether the given quadratic function has a maximum value or a minimum value, and then find the value.

$$y = -x^2 - 10x - 3$$

28. If  $f(x) = \frac{2x-a}{x-3}$  and  $f(4) = 0$ , what is the value of  $a$ ? Where is  $f$  not defined?

29. Write the partial fractions decomposition of  $f(x) = \frac{x}{x^2 - 5x + 6}$

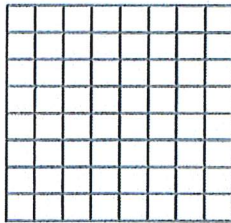
30. Write the following expression as single quotient in which only positive exponents appear:

$$(x^2 + 1)^{1/2} + x \cdot \frac{1}{2}(x^2 + 1)^{-1/2} \cdot 2x$$

31. Factor:  $4x^{1/3}(2x + 1) + 2x^{4/3}$

32. Use the **signum function**. The signum function is defined by  $\text{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ .

Sketch the graph of  $\text{sgn}(x)$ .



Find the following limits (if possible):

a)  $\lim_{x \rightarrow 0^-} \text{sgn}(x)$

b)  $\lim_{x \rightarrow 0^+} \text{sgn}(x)$

c)  $\lim_{x \rightarrow 0} \text{sgn}(x)$

33. Find the limit:  $\lim_{x \rightarrow 2} (3x^2 + 6)$

34. Find the limit:  $\lim_{x \rightarrow 1} f(x)$ , if  $f(x) = \begin{cases} x^2 + 4, & x \neq 1 \\ 2, & x = 1 \end{cases}$



35. Find the horizontal asymptote (if any) for  $f(x) = \frac{ax^3}{b + cx + dx^2}$

36. Find the horizontal asymptote (if any):  $f(x) = \frac{2x-3}{x+2} + \frac{3}{x-5}$

37. Find the value(s) of  $x$  for which  $f(x) = \frac{2x+6}{x^2-9}$  is discontinuous and label these discontinuities.

38. Determine the intervals on which  $f(x)$  is continuous,  $f(x) = \tan(0.5x)$

39. Let  $f(x) = \frac{1}{|x|}$  and  $g(x) = x - 1$ . Find all the values of  $x$  for which  $f(g(x))$  is discontinuous.

40. Find all vertical asymptotes of  $f(x)$  if  $f(x) = \frac{-3x+3}{(x-1)(x^2+x-1)}$



41. Find the inverse of  $f(x) = x^2 - 2$  for  $x \geq 0$ , and call it  $g(x)$ . Graph both  $f(x)$  and  $g(x)$ . Are they symmetric? If so, about what?

42. Give an example of a function that does not have an inverse.

43. Is  $f(x) = \sin x$  an even or an odd function (or neither)? Why?

44. Is  $f(x) = \cos x$  an even or an odd function (or neither)? Why?

45. Is  $f(x) = x^3 + 1$  an even or an odd function (or neither)? Why?